CHAPTER # 3 BLOCK DIAGRAM

After completing this chapter, the students will be able to:

- Find the transfer function of electrical circuits,
- Reduce a block diagram of multiple subsystems to a single block representing the transfer function from input to output (Block diagram algebra).
- Apply block-diagram algebra to Single Input Single Output (SISO), Multi Input Single Output (MISO) and Multi Input Multi Output (MIMO) systems.

1. Introduction

In the previous chapter, we defined the Laplace transform and its inverse. We presented the idea of the partial-fraction expansion and applied the concepts to the solution of differential equations.

Consider a control system that shown in Fig. 1:

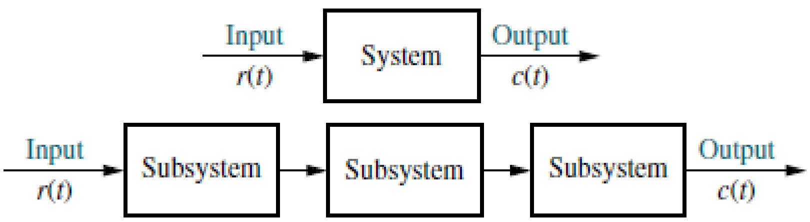


Fig. 1 Single, or multiple, block diagram representation

Now, we are ready to formulate the above system by establishing a viable definition for a function that algebraically relates a system's output to its input. Unlike the differential equation, the function allows us to algebraically combine mathematical representations of subsystems to yield a total system representation.

The transfer function can be represented as a block diagram, as shown in Fig. 2, with the input R(s) to the left, the output C(s) to the right, and the system transfer function G(s) inside the block.

Fig. 2 Single block diagram representation

$$C(s) = R(s).G(s)$$

<u>Example (1):</u>

Find the system Transfer function given by the following D.E.:

$$\frac{dc(t)}{dt} + 2c(t) = r(t)$$

Taking the Laplace transform of both sides, assuming zero initial conditions, we have

$$sC(s) + 2C(s) = R(s)$$

Then the system Transfer Function G(s) is:

$$G(s) = \frac{C(s)}{R(s)} = \frac{1}{s+2}$$

To obtain the system response C(s) at unit-step input R(s), then:

$$C(s) = R(s)G(s) = \frac{1}{s(s+2)}$$

Expanding by partial fractions, we get

$$C(s) = \frac{1/2}{s} - \frac{1/2}{s+2}$$

Finally, taking the inverse Laplace transform of each term yields

$$c(t) = \frac{1}{2} - \frac{1}{2}e^{-2t}$$

2. Transfer Function of Electric Circuits:

Consider the RLC circuit given in Fig. 3, find T.F. assuming the voltage V_c is the circuit output.

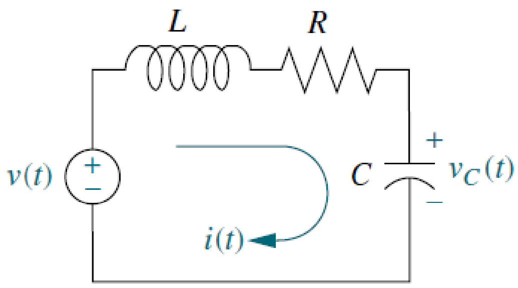


Fig. 3, RLC circuit

Using mesh analysis:

$$L\frac{di(t)}{dt} + Ri(t) + \frac{1}{C}\int_0^t i(\tau)d\tau = v(t)$$

$$L.S.I(s) + R.I(s) + \frac{1}{C}.\frac{I(s)}{s} = V(s)$$

$$I(s)\left\{LS + R + \frac{1}{Cs}\right\} = V(s)$$

$$I(s)\left\{\frac{LCs^2 + RCs + 1}{Cs}\right\} = V(s)$$

The circuit output $V_c(t)$ is given by $\frac{1}{C}\int_0^t i(\tau)d\tau$

$$\frac{I(s)}{Cs} = V_c(s)$$

Then the circuit T.F. is given by:

$$\frac{V(s)}{V_c(s)} = G(s) = \frac{1}{LCs^2 + RCs + 1}$$

Please refer to the table given below to simulate simple electric circuits

Component	Voltage-current	Current-voltage	Impedance $Z(s) = V(s)/I(s)$	Admittance $Y(s) = I(s)/V(s)$
Capacitor	$v(t) = \frac{1}{C}\int_0^1 i(\tau)d\tau$	$i(t) = C\frac{dv(t)}{dt}$	$\frac{1}{Cs}$	Cs
Resistor	$v(t) = Ri(t)$	$i(t) = \frac{1}{R}v(t)$	R	$\frac{1}{R} = G$
Inductor	$v(t) = L\frac{di(t)}{dt}$	$i(t) = \frac{1}{L}\int_0^1 v(\tau)d\tau$	Ls	$\frac{1}{Ls}$

Example (2):

Consider a more complicated circuit as shown in Fig. 4. Find the T.F. $I_2(s)/V(s)$

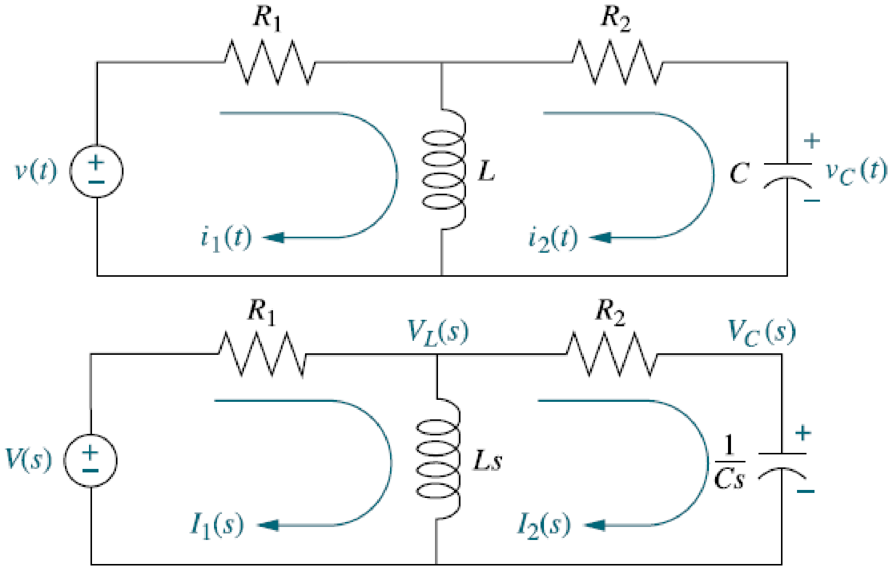


Fig. 4, RLC circuit

For Mech (1):

$$V(s) = R_1 I_1(s) + LsI_1(s) - LsI_2(s)$$

For Mech (2):

$$LsI_2(s) + R_2 I_2(s) + \frac{1}{Cs} I_2(s) - LsI_1(s) = 0$$

$$\begin{aligned} (R_1 + Ls)I_1(s) \qquad\qquad & - LsI_2(s) = V(s) \\ - LsI_1(s) + \left(Ls + R_2 + \frac{1}{Cs}\right) & I_2(s) = 0 \end{aligned}$$

$$\Delta = \begin{vmatrix} (R_1 + Ls) & -Ls \\ -Ls & \left(Ls + R_2 + \frac{1}{Cs}\right) \end{vmatrix}$$

Using Cramer's rule:

$$I_2(s) = \frac{\begin{vmatrix} (R_1 + Ls) & V(s) \\ -Ls & 0 \end{vmatrix}}{\Delta} = \frac{LsV(s)}{\Delta}$$

$$\frac{I_2(s)}{V(s)} = \frac{Ls}{\Delta} = \frac{LCs^2}{(R_1 + R_2)LCs^2 + (R_1 R_2 C + L)s + R_1}$$

3. Operational Amplifiers (Op. Amp.)

For the operational amplifier, shown in Fig. 5, the differential input is v_2 -v_1,

If v_2 is grounded, the amplifier is called inverting op. amp.

In circuit (a), the output V_o is given by: $v_o(t) = -Av_1(t)$

In circuit (b), the T.F. is given by:

$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}$$

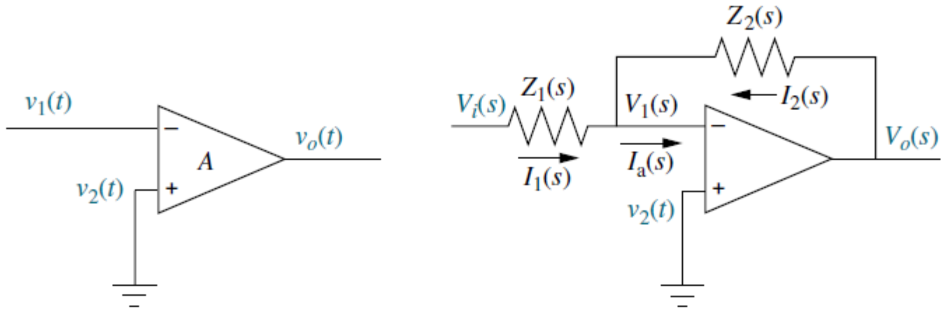


Fig. 5, Inverting Op. Amp.

Example (3):

Find the T.F. for the Op. Amp. Circuit shown in Fig. 6.

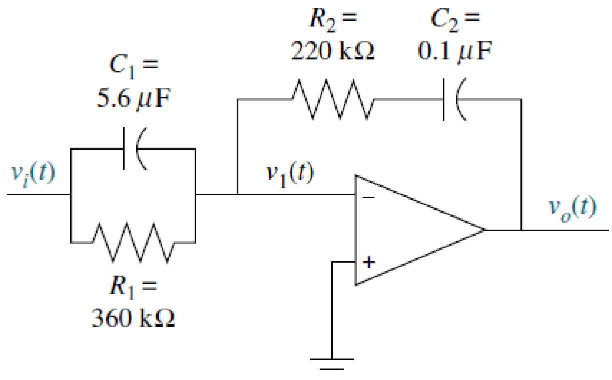


Fig. 6, Op. Amp. circuit

$$Z_1(s) = \frac{1}{C_1 s + \frac{1}{R_1}} = \frac{1}{5.6\times 10^{-6}s + \frac{1}{360\times 10^3}} = \frac{360\times 10^3}{2.016s+1}$$

$$Z_2(s) = R_2 + \frac{1}{C_2 s} = 220\times 10^3 + \frac{10^7}{s}$$

$$\frac{V_o(s)}{V_i(s)} = -1.232\frac{s^2 + 45.95s + 22.55}{s}$$

For Non-inverting op. amp., shown in Fig. 7, the T.F. is given by:

$$\frac{V_o(s)}{V_i(s)} = \frac{Z_1(s) + Z_2(s)}{Z_1(s)}$$

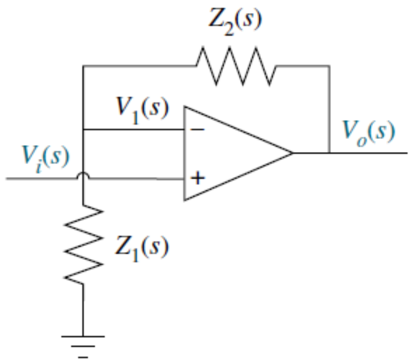


Fig. 7, Non-Inverting Op. Amp.

Example (4):

For the non-inverting Op. Amp. given in Fig. 8, find the circuit T.F.

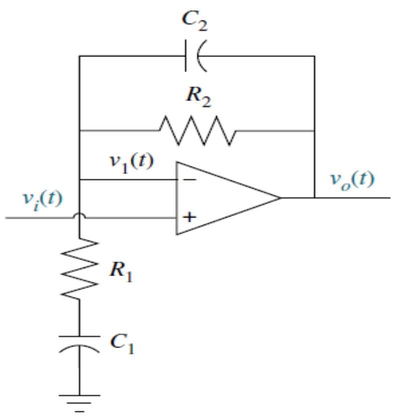


Fig. 8, Non-Inverting Op. Amp.

$$Z_1(s) = R_1 + \frac{1}{C_1 s}$$

$$Z_2(s) = \frac{R_2(1/C_2 s)}{R_2 + (1/C_2 s)}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{C_2C_1R_2R_1s^2 + (C_2R_2 + C_1R_2 + C_1R_1)s + 1}{C_2C_1R_2R_1s^2 + (C_2R_2 + C_1R_1)s + 1}$$

In general, the block diagram consists of blocks, arrows, take (pick) off points and/or summing points. Fig. 9 shows these elements of the block diagram.

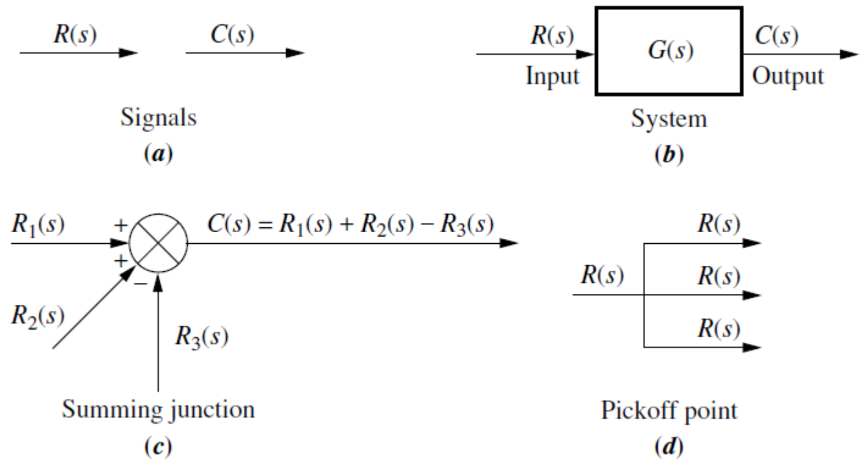


Fig. 9, Basic elements of block diagram

4. Terminology

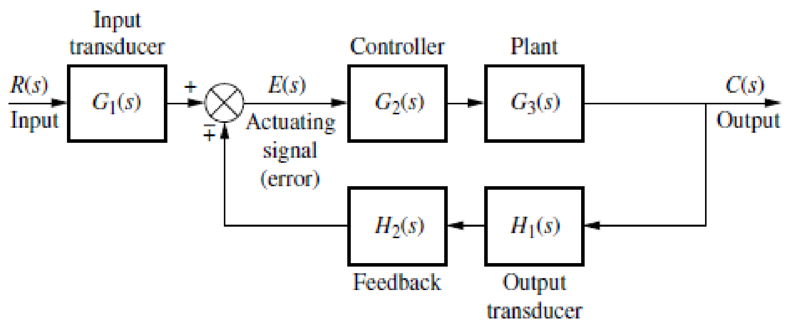


Fig. 9, Block diagram components

Regarding the closed-loop control system shown in Fig. 9, we can define the following terms;

Plant: A physical object to be controlled. The Plant G_3 (s), is the controlled system, of which a particular quantity or condition is to be controlled.

Feedback Control System (Closed-loop Control System): A system which compares output to some reference input and keeps output as close as possible to this reference.

Open-loop Control System: Output of the system is not feedback to the system.

Control Element $G_2(s)$, also called the ***controller***, are the components required to generate the appropriate control signal M (s) applied to the plant

***Feedback Element** $H(s)$* is the component required to establish the functional relationship between the primary feedback signal $B(s)$ and the controlled output $C(s)$.

***Reference Input** $R(s)$* is an external signal applied to a feedback control system in order to command a specified action of the plant. It often represents ideal plant output behavior.

***Controlled Output** $C(s)$* is that quantity or condition of the plant which is controlled

***Actuating Signal** $E(s)$*, also called *the error* or *control action*, is the algebraic sum consisting of the reference input $R(s)$ plus or minus (usually minus) the primary feedback $B(s)$.

***Manipulated Variable** $M(s)$* (*control signal*) is that quantity or condition which the control elements $G_2(s)$ apply to the plant $G_3(s)$.

Forward Path is the path from the actuating signal $E(s)$ to the output $C(s)$.

Feedback Path is the path from the output $C(s)$ to the feedback signal $B(s)$.

Summing Point: A circle with a cross is the symbol that indicates a summing point. The (+) or (−) sign at each arrowhead indicates whether that signal is to be added or subtracted.

Branch (pick/take off) Point: A branch point is a point from which the signal from a block goes concurrently to other blocks or summing points.

We can conclude the above information by the following definitions:

According to the control system shown in Fig 10;

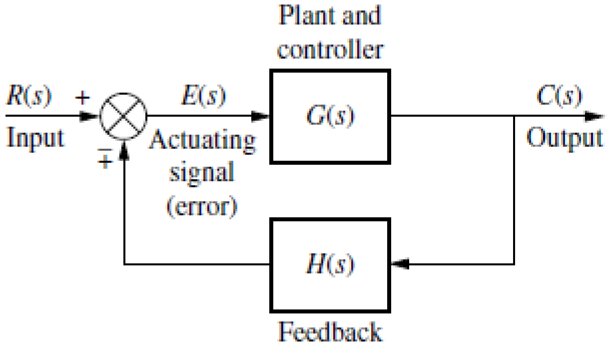


Fig. 10, Block diagram of a closed-loop system with a feedback element.

- $G(s)$ ≡ Direct transfer function = Forward transfer function.
- $H(s)$ ≡ Feedback transfer function.
- $G(s)H(s)$ ≡ Open-loop transfer function.
- $C(s)/R(s)$ ≡ Closed-loop transfer function = Control ratio
- $C(s)/E(s)$ ≡ Feed-forward transfer function.

5. Block Diagrams & Their Simplification

5.1 Cascade (Series) Connection

Figure 11(a) shows an example of cascaded subsystems. Intermediate signal values are shown at the output of each subsystem. Each signal is derived from the product of the input times the transfer function. The equivalent transfer function shown in Fig. 11(b), is the output Laplace transform divided by the input Laplace which is the product of the subsystems' transfer functions.

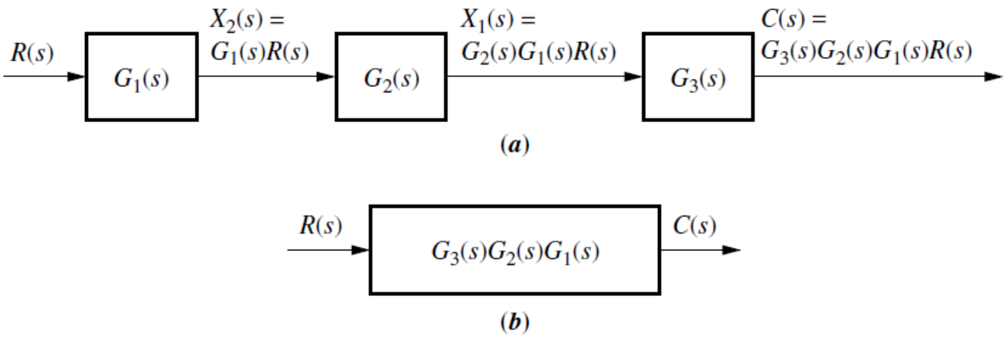


Fig. 11, (a) Original Block Diagram (b) Equivalent Block Diagram

5.2 Parallel Connection

Figure 12 (a) shows an example of parallel subsystems. Again, by writing the output of each subsystem, we can find the equivalent transfer function. Parallel subsystems have a common input and an output formed by the algebraic sum of the outputs from all of the subsystems. The equivalent transfer function is given in Fig. 12(b):

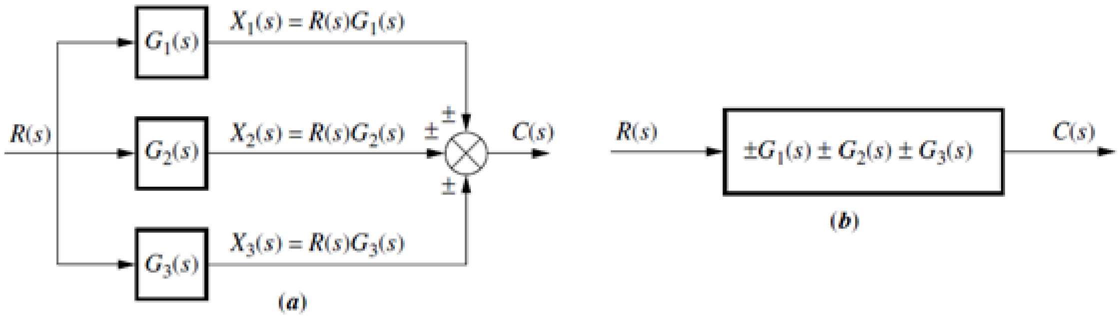


Fig. 12, (a) Original Block Diagram (b) Equivalent Block Diagram

5.3 Feedback Connections

The third connection is the feedback form as shown in Fig. 13 (a). The feedback forms the basis for our study of control systems engineering.

We know that $C(s) = G(s)\,E(s)$ & $B(s) = H(s)C(s)$

Where $E(s) = R(s) \mp B(s) = R(s) \mp H(s)C(s)$

Eliminating $E(s)$ from these equations gives

$C(s) = G(s)\,[R(s) \mp H(s)C(s)]$ This can be written in the form

$[1 \pm G(s)\,H(s)]\,C(s) = G(s)\,R(s)$

The equivalent transfer function is given in Fig. 13(b):

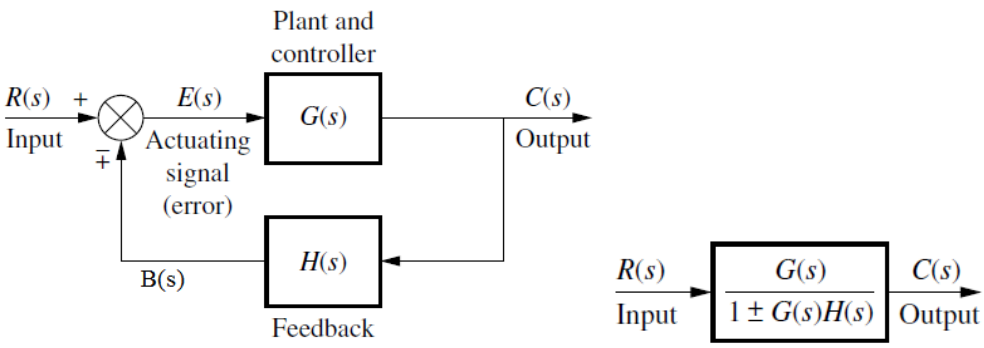
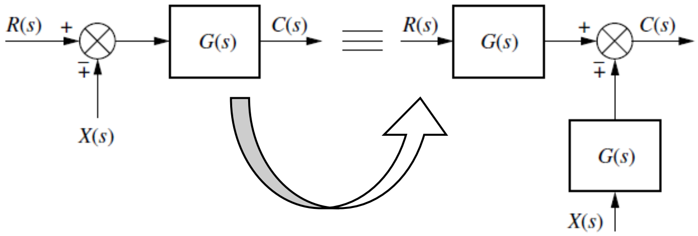


Fig. 13, (a) Feedback connection (b) Equivalent block

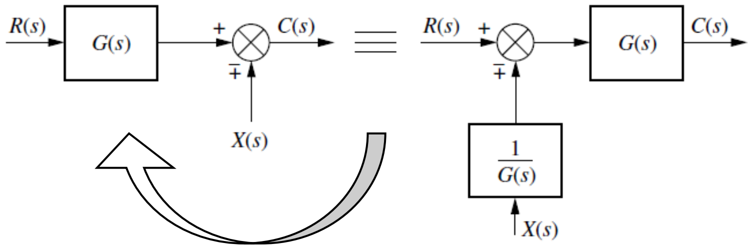
The ***Characteristic equation*** of the system is defined as an equation obtained by *setting the denominator polynomial of the transfer function to zero*. The ***Characteristic equation*** for the above system is:

$$1 + G(s)H(s) = 0$$

5.4 Moving Summing point to get known connection

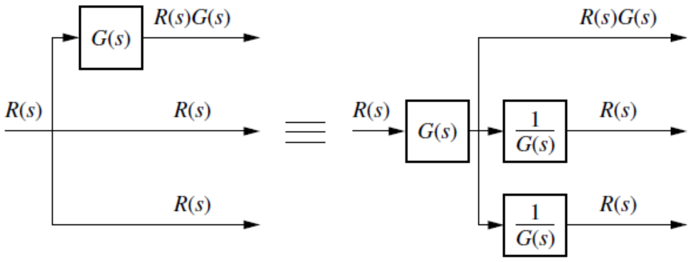


Moving summing point and jump over a block in the direction of the forward path, we must multiply with the jumped block.

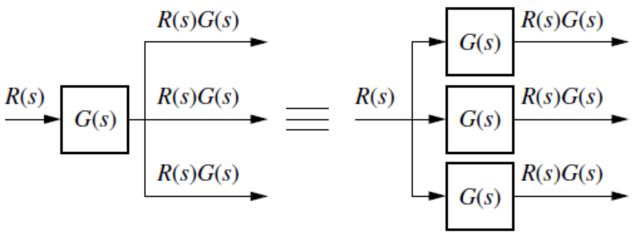


Moving summing point and jump over a block in the direction of the feedback path, we must divide by the jumped block.

5.5 Moving Pick/take off point to get known connection



Moving take off point and jump over a block in the direction of the forward path, we must divide by the jumped block.



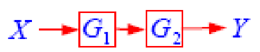
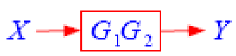
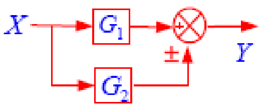
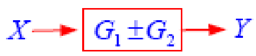
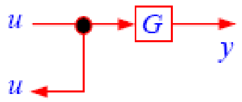
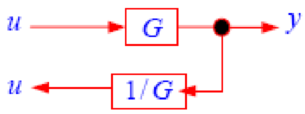
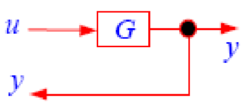
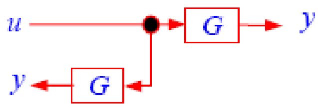
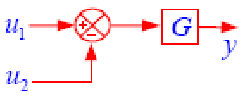
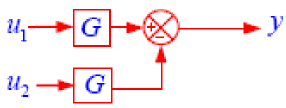
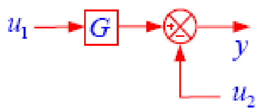
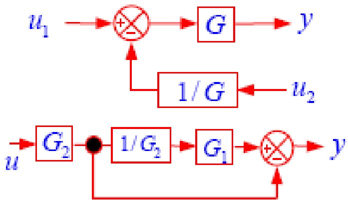
Moving take off point and jump over a block in the direction of the feedback path, we must multiply with the jumped block.

6. Block Diagram Reduction Rules

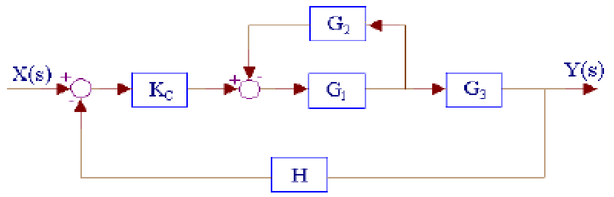
In many practical situations, the block diagram of a Single Input-Single Output (SISO), feedback control system may involve several feedback loops, summing points and/or take off points. In principle, the block diagram of (SISO) closed loop system, no matter how complicated it is, it can be reduced to the standard single loop form (Canonical form) shown in Fig. 13. The basic approach to simplify a block diagram can be summarized in the following Table;

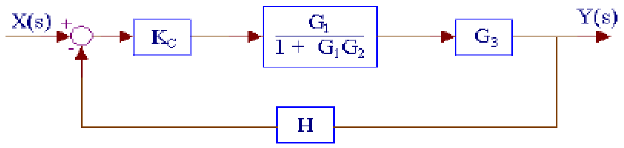
1.	Combine all cascade blocks
2.	Combine all parallel blocks
3.	Eliminate all minor (interior) feedback loops
4.	Shift summing points to left
5.	Shift take off points to the right
6.	Repeat Steps 1 to 5 until the canonical form is obtained

6.1. Some Basic Rules with Block Diagram Transformation

	Manipulation	Original Block Diagram	Equivalent Block Diagram	Equation
1	Combining Blocks in Cascade			$Y=(G_1G_2)X$
2	Combining Blocks in Parallel; or Eliminating a Forward Loop			$Y=(G_1\pm G_2)X$
3	Moving a pickoff point behind a block			$y=Gu$ $u=\frac{1}{G}y$
4	Moving a pickoff point ahead of a block			$y=Gu$
5	Moving a summing point behind a block			$e_2=G(u_1-u_2)$
6	Moving a summing point ahead of a block			$y=Gu_1-u_2$ $y=(G_1-G_2)u$

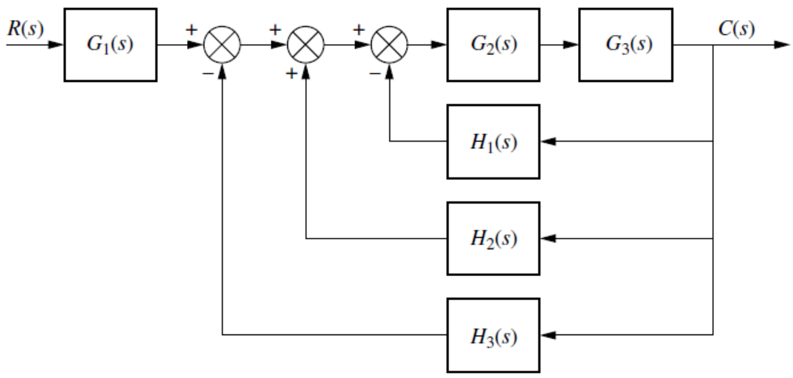
Example (5):

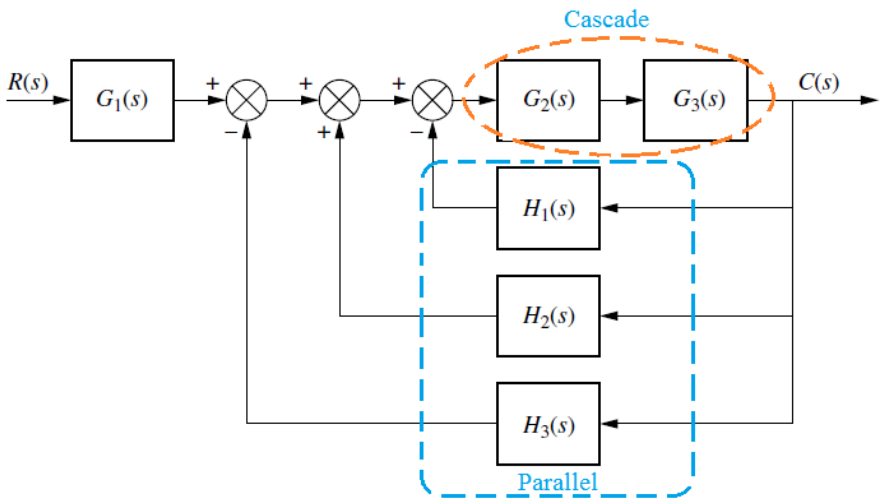


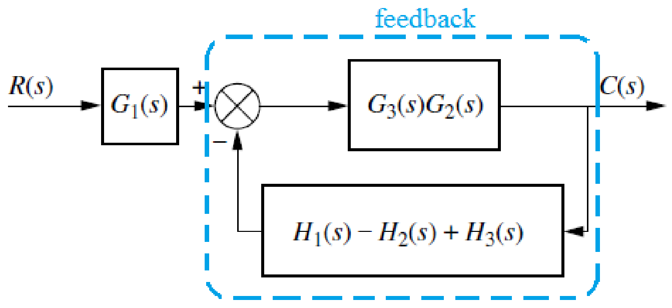


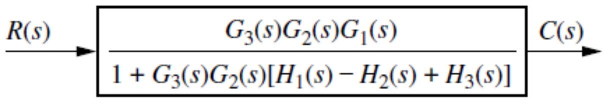
Example (6):

Reduce the given block diagram to a single block form.



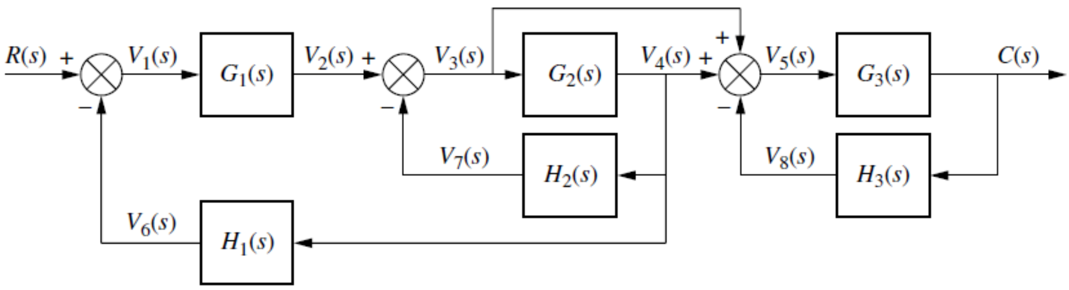


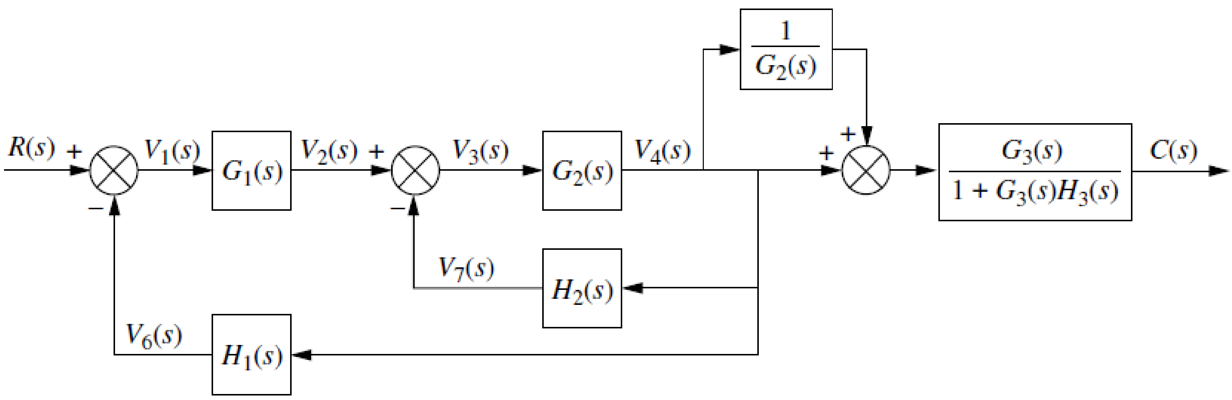


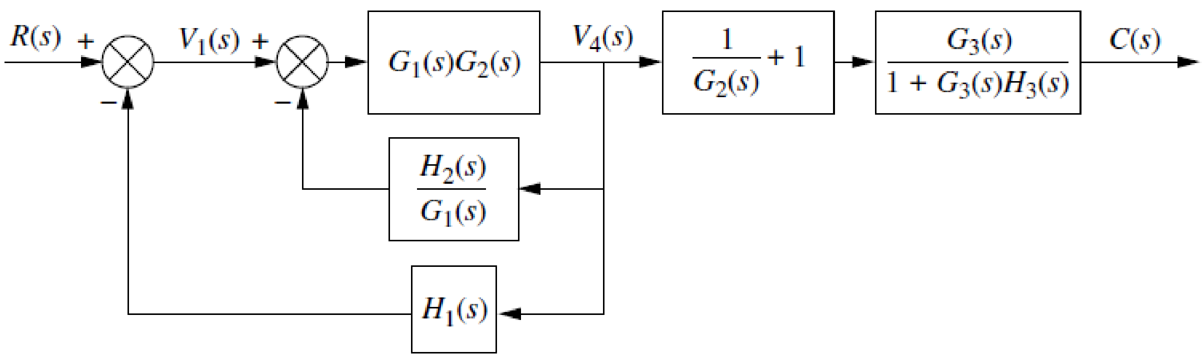


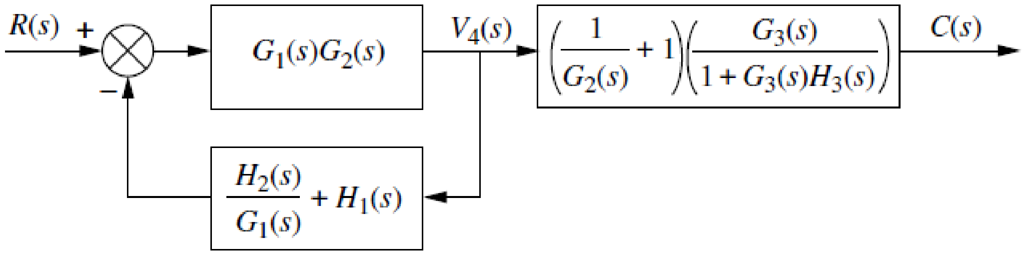
Example (7):

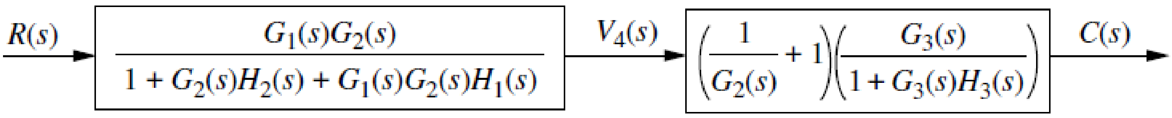
The main problem here is the feed-forward of V3(s). Solution is to move this pickoff point forward.

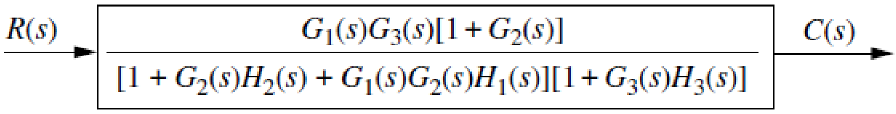




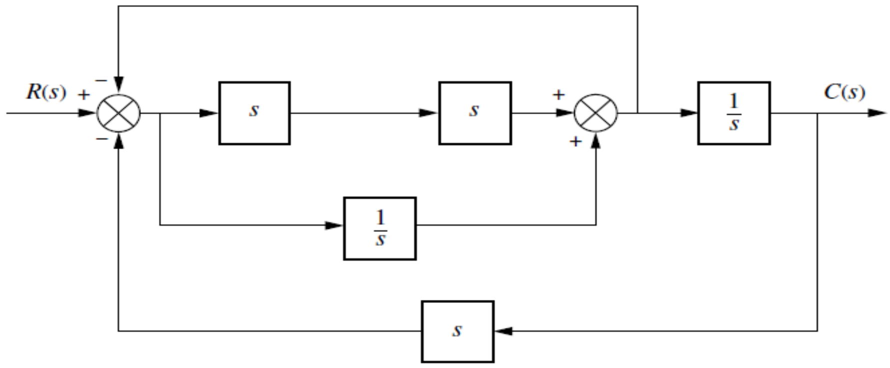


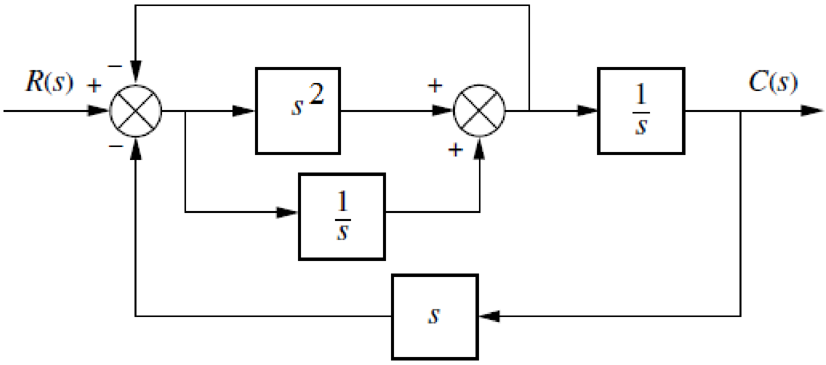

R(s)
+
−
G1(s)G2(s)
V4(s)
(1/G2(s) + 1)(G3(s)/(1 + G3(s)H3(s)))
C(s)
H2(s)/G1(s) + H1(s)

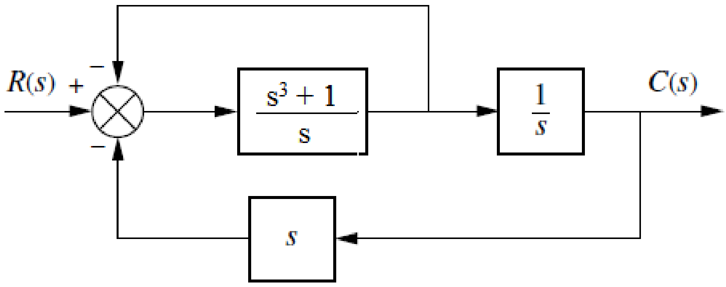

R(s)
G1(s)G2(s) / (1 + G2(s)H2(s) + G1(s)G2(s)H1(s))
V4(s)
(1/G2(s) + 1)(G3(s)/(1 + G3(s)H3(s)))
C(s)

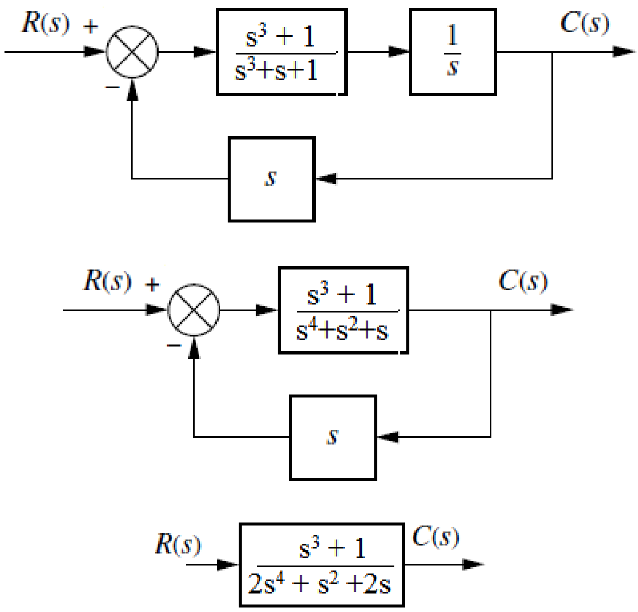

R(s)
G1(s)G3(s)[1 + G2(s)] / [1 + G2(s)H2(s) + G1(s)G2(s)H1(s)][1 + G3(s)H3(s)]
C(s)

Example (8):


R(s)
C(s)
s
s
1/s
1/s
s

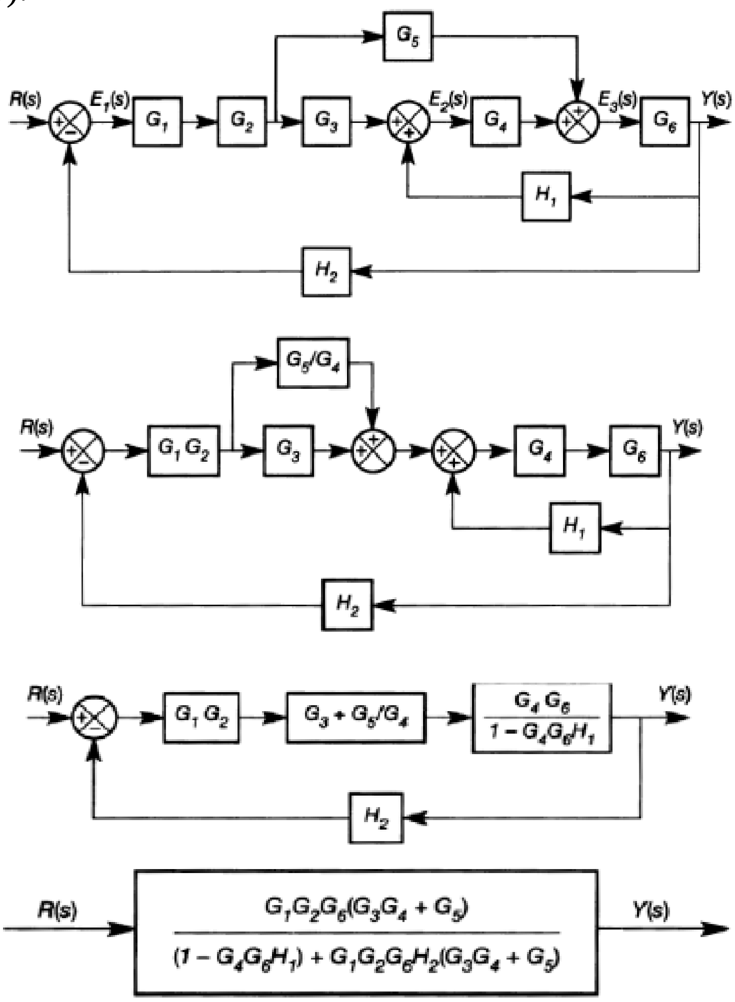

R(s)
C(s)
s 2
1/s
1/s
s


R(s)
C(s)
(s³ + 1)/s
1/s
s



Example (9):

Use block diagram reduction to simplify the block diagram below into a single block relating $Y(s)$ to $R(s)$.

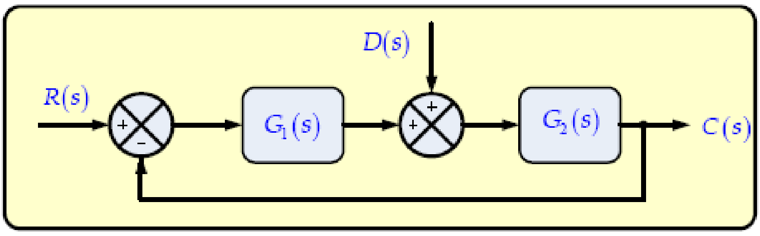


7. Multiple-Inputs cases

In feedback control system, we often encounter multiple inputs to represent a disturbance or something else. For a linear system, we can apply the ***superposition*** principle to solve this type of problems, i.e. to treat each input one at a time while setting all other inputs to zeros, and then algebraically add all the outputs as follows:

1.	Set all inputs to zero except one
2.	Transform the block diagram to solvable form
3.	Find the output response due to the chosen input action alone
4.	Repeat Steps 1 to 3 for each of the remaining inputs
5.	Algebraically sum all the output responses obtained in Step 3

Example (10): Determine the output C(S) of the following system



Using the superposition principle, the procedure is illustrated in the following steps:

Step1: Put $D(s) \equiv 0$ as shown in Fig. (a).

Step2: Reduce The block diagrams to the block shown in Fig. (b)

Step 3: The output C_R due to input $R(s)$ is shown in Fig. (c) and is given by the relationship

$$C_R = \frac{G_1 G_2}{1 + G_1 G_2} \cdot R$$

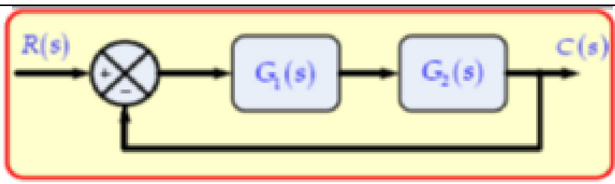


Figure (a)

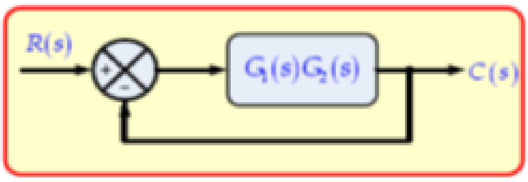


Figure (b)

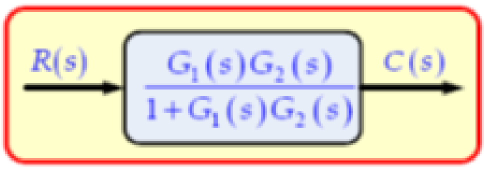


Figure (c)

Step 4: Put $R(s) \equiv 0$ as shown in Fig. (d).

Step 5: Put -1 into a block, representing the negative feedback effect as shown in Fig. (d)

Step 6: Rearrange the block diagrams as shown in Fig. (e).

Step 7: Let the -1 block be absorbed into the summing point as shown in Fig. (f).

Step 8: The output C_D due to input $D(S)$ is :

$$C_D = \frac{G_2}{1 + G_1 G_2} D(s)$$

The total output is C:

$C(s) = C_R + C_D$

$$C(s) = \frac{G_2}{1 + G_1 G_2} (G_1 R + D)$$

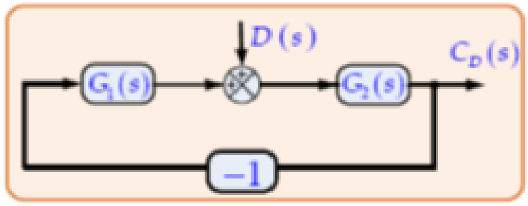


Figure (d)

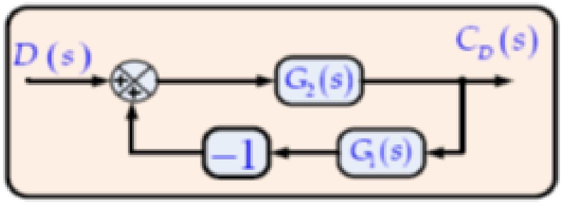


Figure (e)

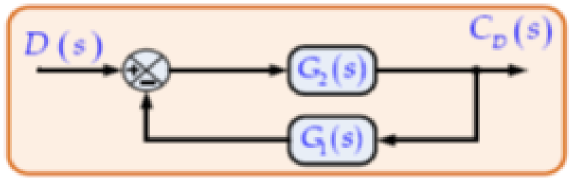
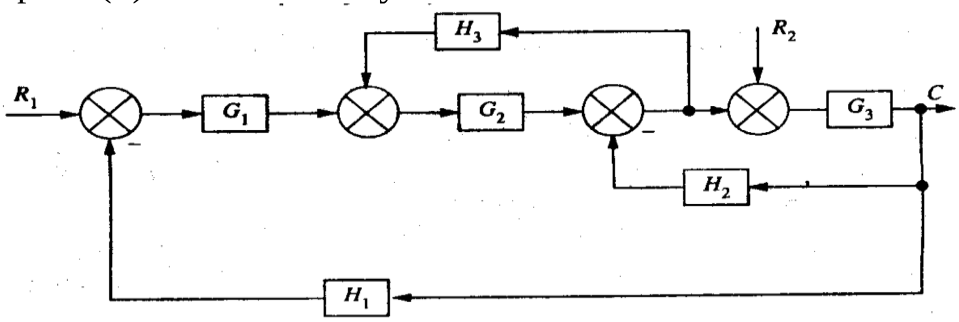


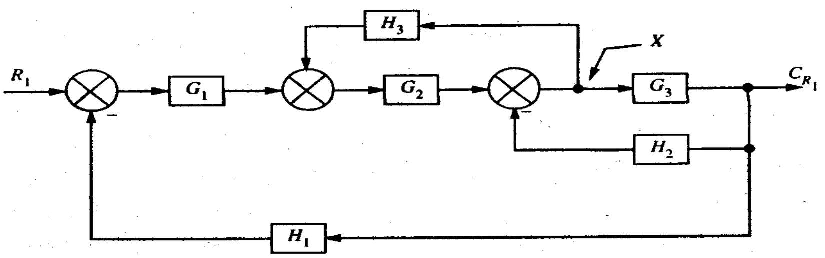
Figure (f)

Example (11):

Find the output C(S) of the control system shown below.

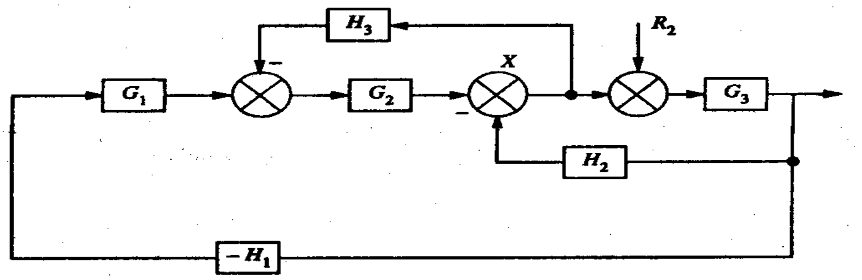


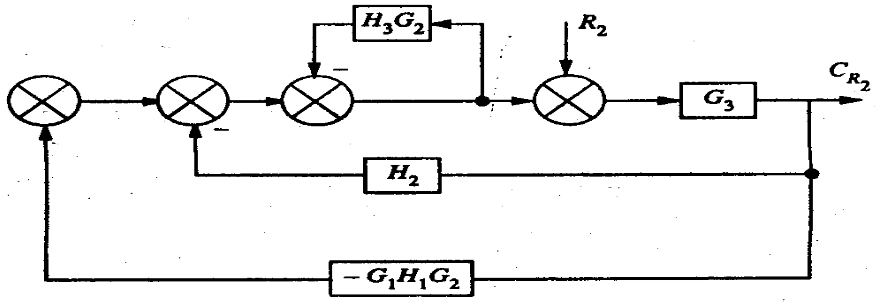
For Input R_1:

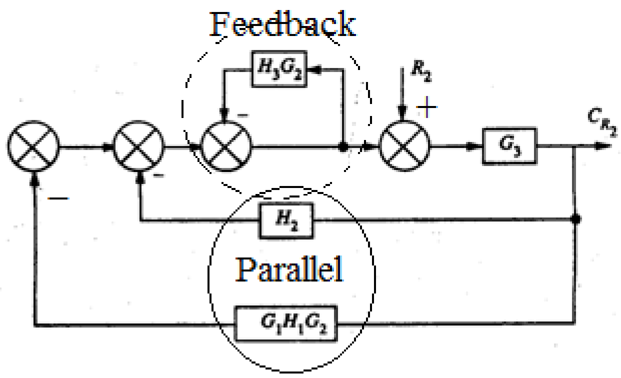


$$C_{R_1} = \left[\frac{G_1 G_2 G_3}{1 - G_3 H_2 + H_3 G_2 + G_1 G_2 G_3 H_1}\right] R_1$$

For input R_2:





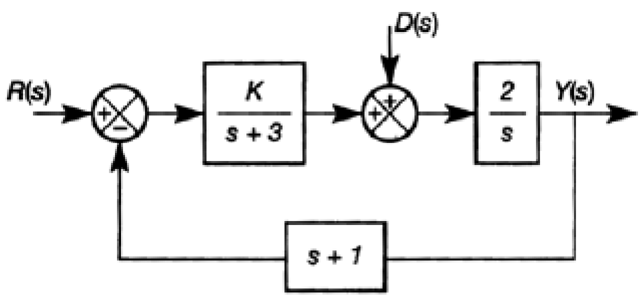


$$C_{R_2} = \left[\frac{G_3\,[1 + G_2 H_3]}{1 - G_2 H_3 + G_3\,[G_1 G_2 H_1 + H_2]}\right] R_2$$

$$C = C_{R_1} + C_{R_2}$$

$$C = \left[\frac{G_1 G_2 G_3}{1 - G_3 H_2 + G_2 H_3 + G_1 G_2 G_3 H_1}\right] R_1 + \left[\frac{G_3\,[1 + G_2 H_3]}{1 - G_2 H_3 + G_3\,[G_1 G_2 H_1 + H_2]}\right] R_2$$

<u>Example (12):</u>



Setting $D(s) = 0$ gives the transfer function between $Y(s)$ and $R(s)$ as

$$\frac{Y(s)}{R(s)} = \frac{2K}{s(s+3) + 2K(s+1)}$$

Setting $R(s) = 0$ gives the transfer function between $Y(s)$ and $D(s)$ as

$$\frac{Y(s)}{D(s)} = \frac{2(s+3)}{s(s+3) + 2K(s+1)}$$

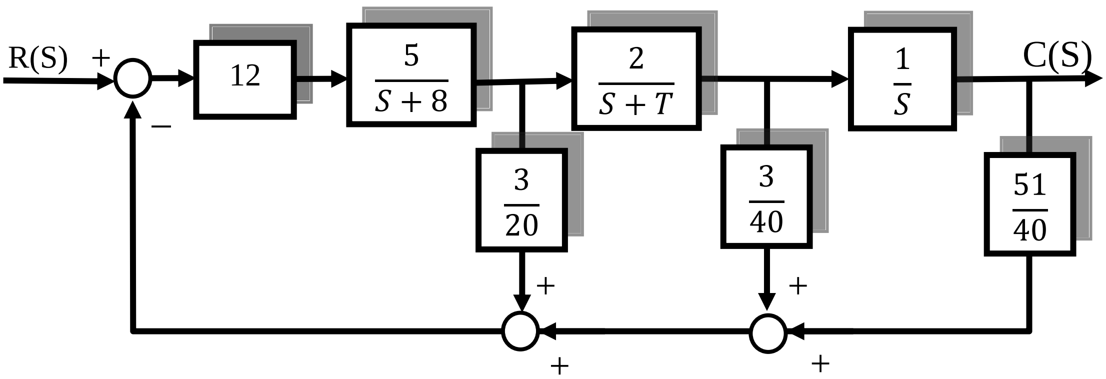
$$Y(s) = \frac{2KR(s) + 2(s+3)D(s)}{s(s+3) + 2K(s+1)}$$

Example (13):

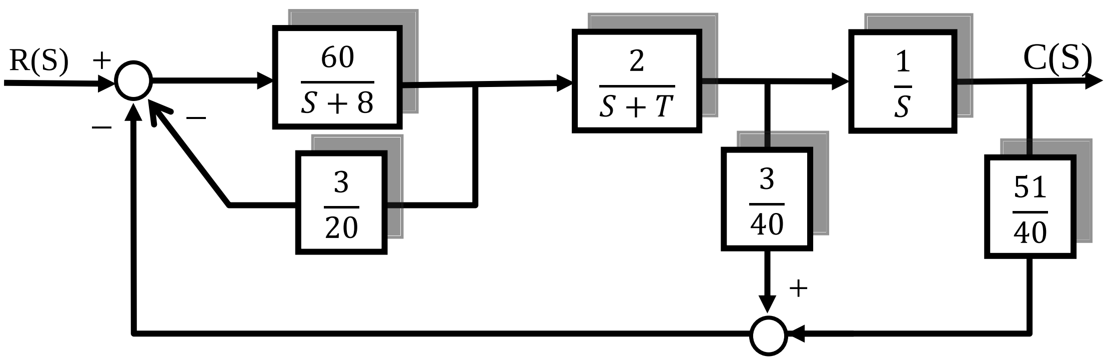
For the closed-loop control system shown below,

a) Using block diagram algebra, find the system transfer function C(S)/R(S).

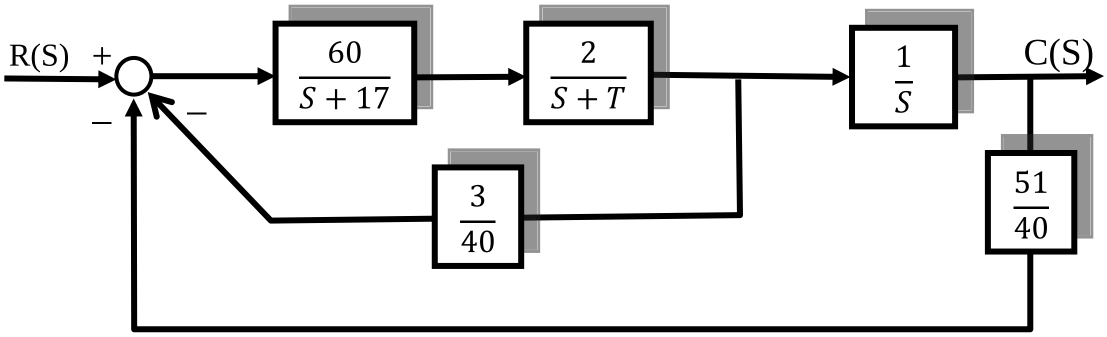
b) Obtain the system characteristic equation.



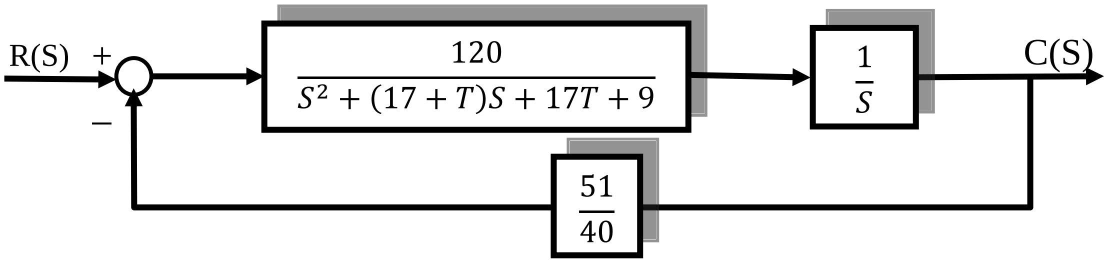
The blocks 12 & 5/(S+8) are cascade

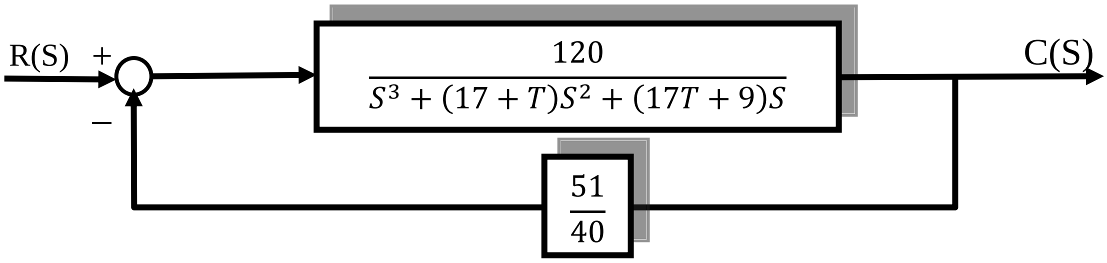


The blocks 60/(S+8) & 3/20 are canonical



The blocks 60/(S+17) cascaded with 2/(S+T) and the result is canonical with 3/40





$$\frac{C(S)}{R(S)} = \frac{120}{S^3 + (17+T)S^2 + (17T+9)S + 153}$$

The system characteristic equation is

$$S^3 + (17+T)S^2 + (17T+9)S + 153 = 0$$

Rearrange the above equation to be:

$$S^3 + 17S^2 + TS^2 + 17TS + 9S + 153 = 0$$

$$S^3 + 17S^2 + 9S + 153 + TS(S+17) = 0$$

Example (14):

If the control systems shown in Fig. A and B are equivalent, Find G_{eq}.

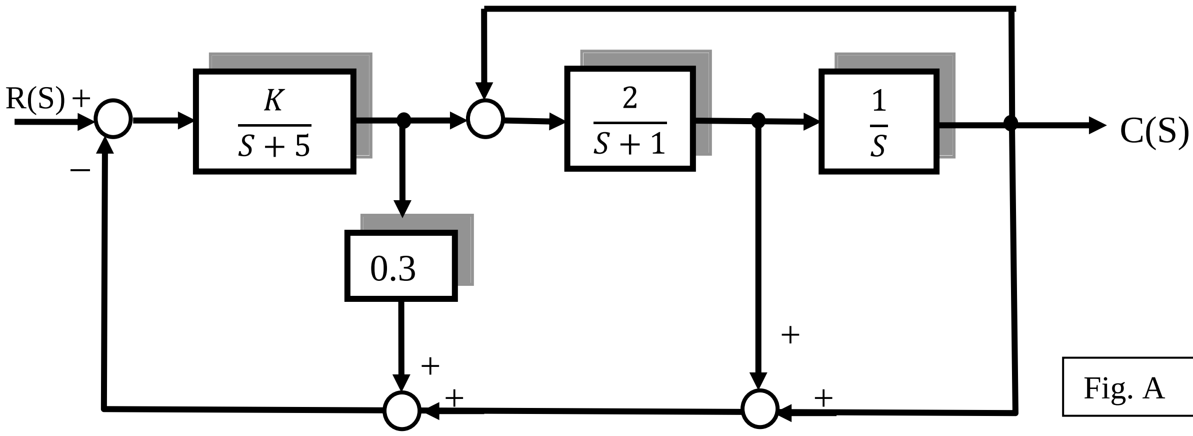


Fig. A

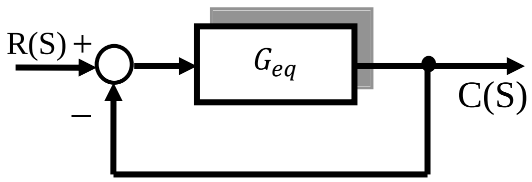
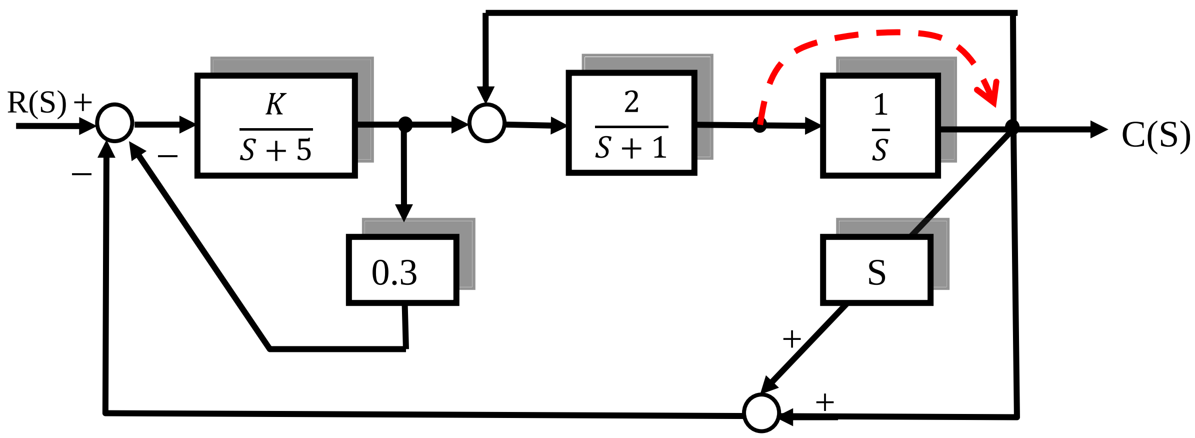
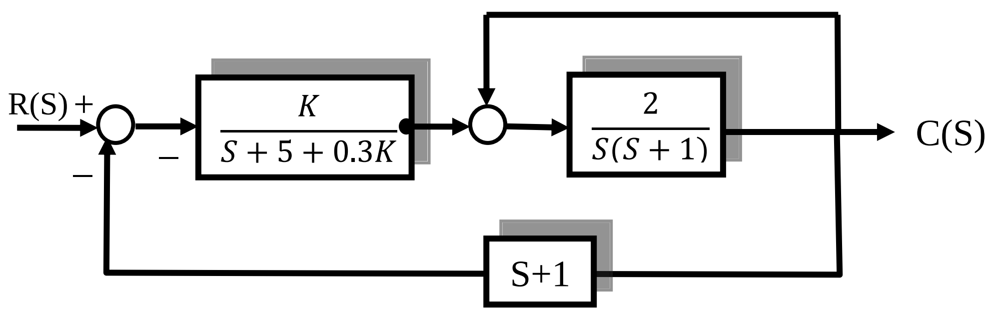
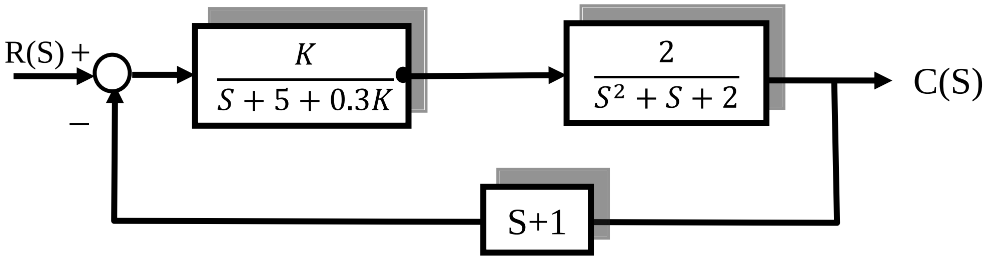


Fig. B

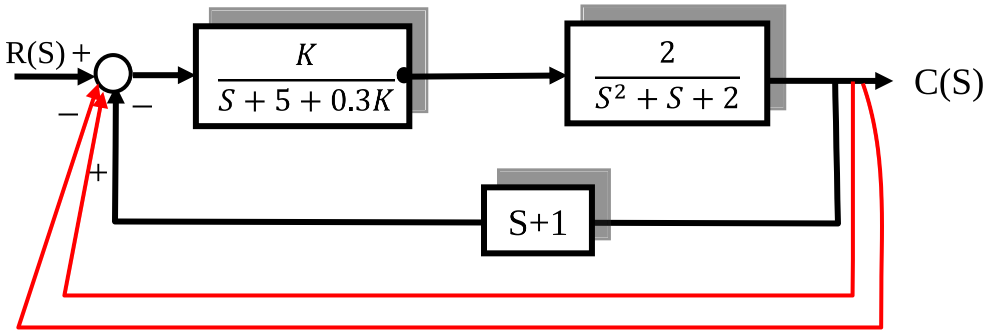
Rearrange the block diagram as follows:

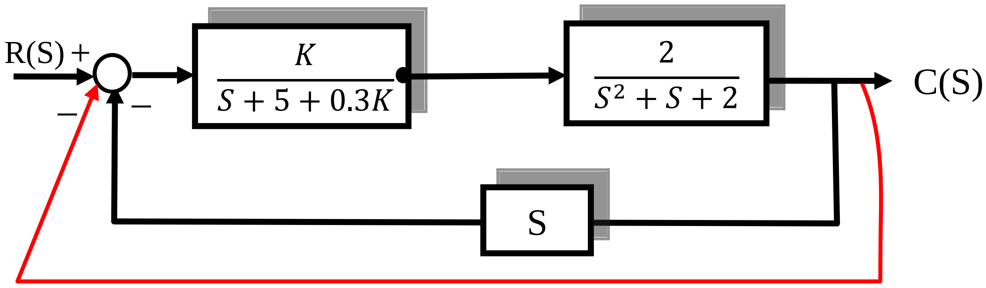


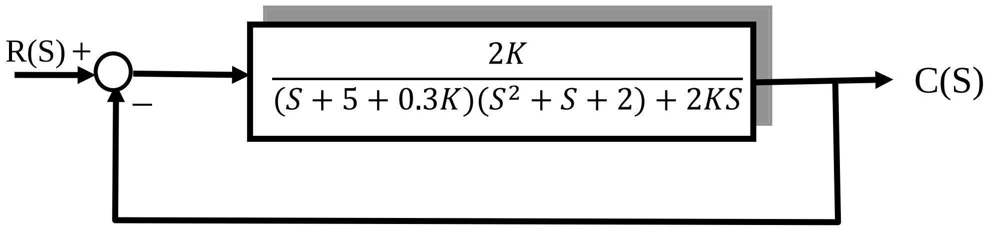




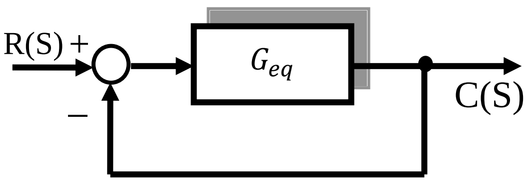
Add unity feedback with negative and positive sign







By comparing with the equivalent block diagram:

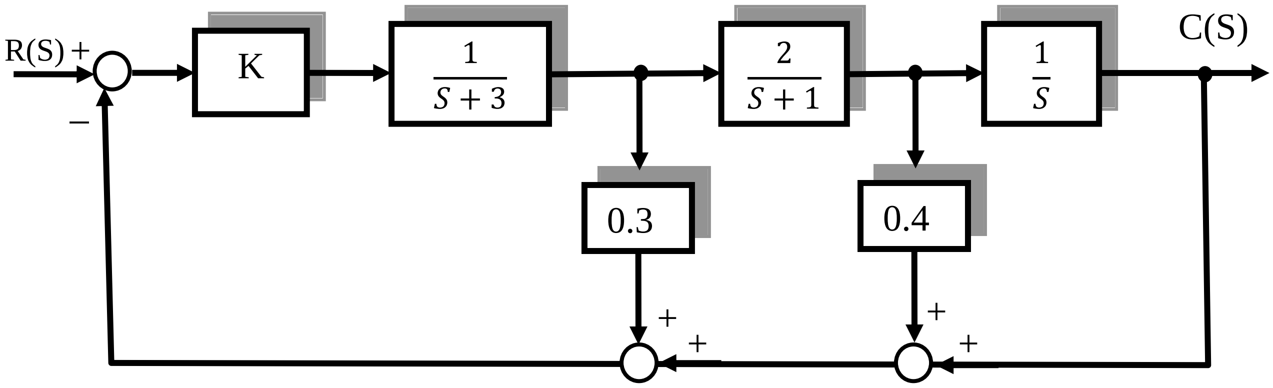


We get that

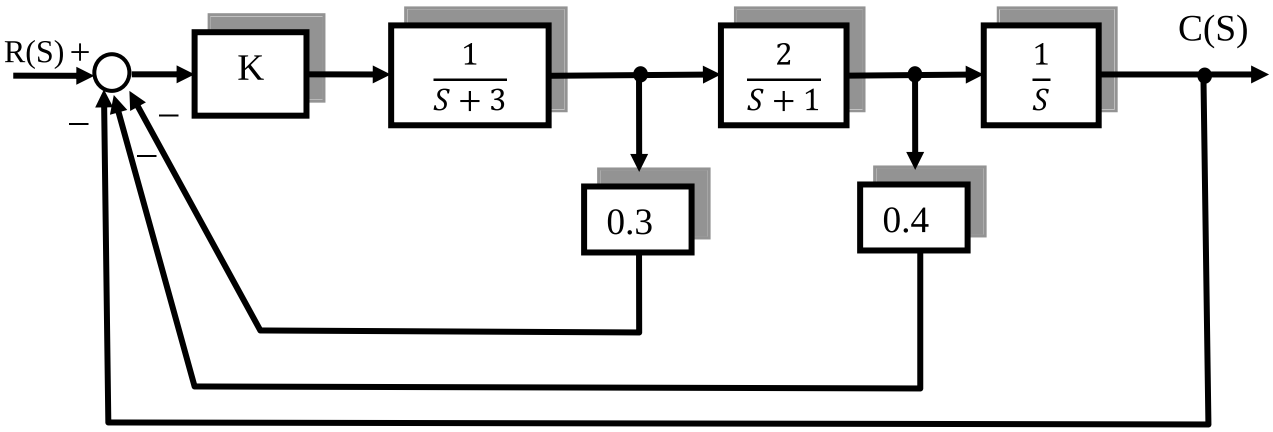
$$G_{eq}(S) = \frac{2K}{(S+5+0.3K)(S^2+S+2)+2KS}$$

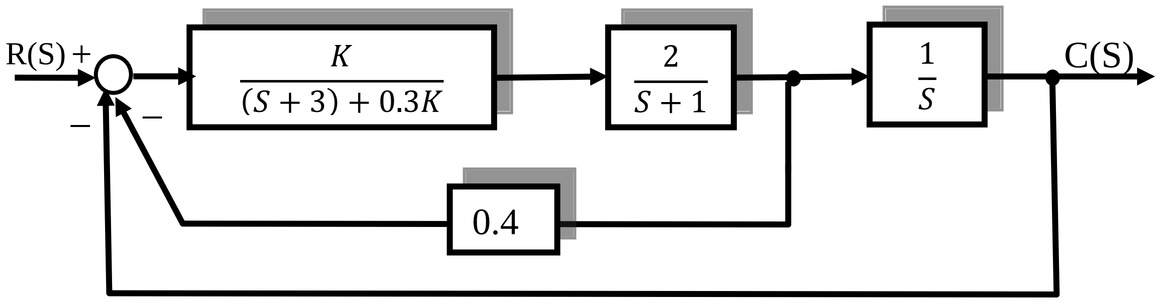
Example (15):

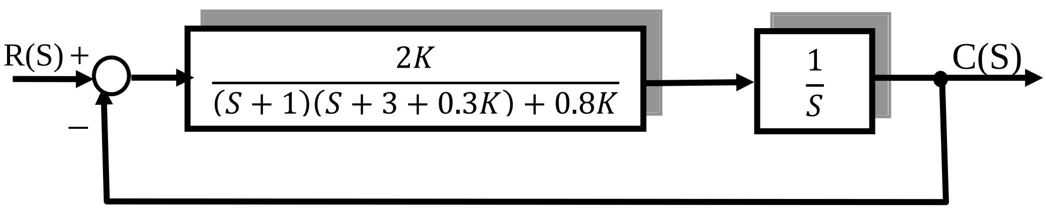
For the control system shown below, Calculate the transfer function C(s)/R(s)



The block diagram can be rearranged as:







Therefore, the closed loop T.F. is:

$$\frac{C(S)}{R(S)} = \frac{2K}{S\{(S+1)(S+3+0.3K)+0.8K\}+2K}$$

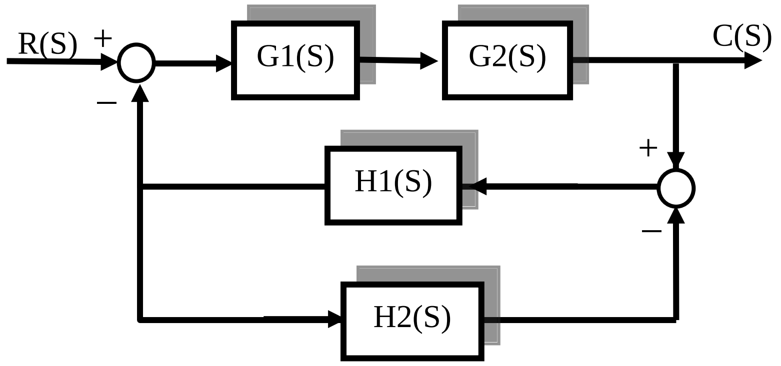
The system characteristic equation is given as:

$$S\{(S+1)(S+3+0.3K)+0.8K\}+2K = 0$$

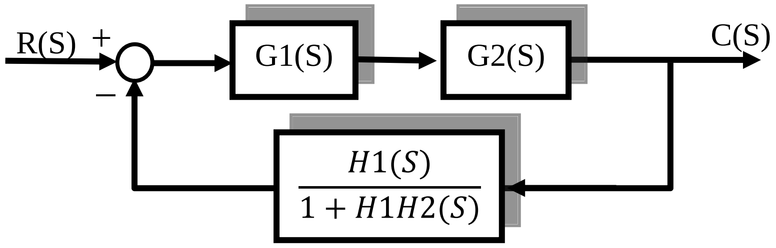
$$S^3 + 4S^2 + 3S + 0.3K(S^2 + 3.667S + 6.667) = 0$$

Example (16):

For the control system shown below, Obtain the transfer function $C(s)/R(s)$.



The blocks H1(S) and H2(S) are canonical and can be simplified as

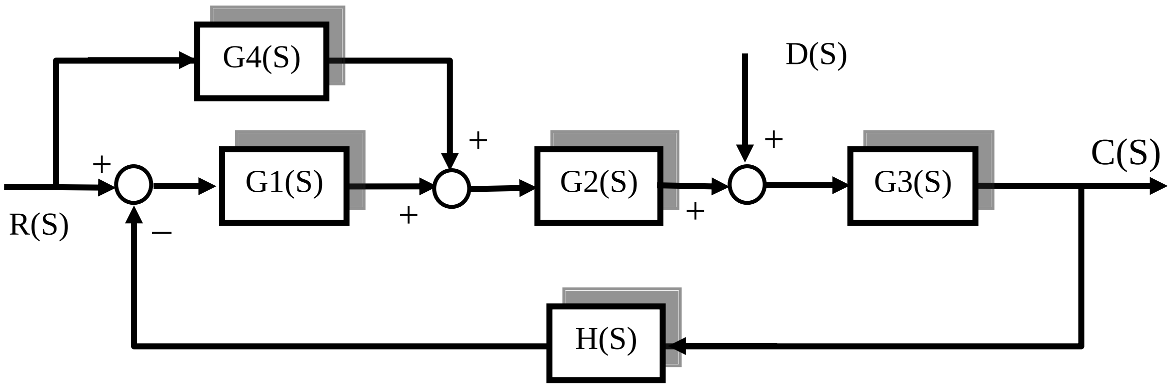


The blocks G1(S) and G2(S) are cascaded and the result is canonical with $\frac{H1(S)}{1+H1H2(S)}$

$$\frac{C(S)}{R(S)} = \frac{G1\,G2\,\{1+H1\,H2\}}{1+H1\,H2+G1\,G2\,H1}$$

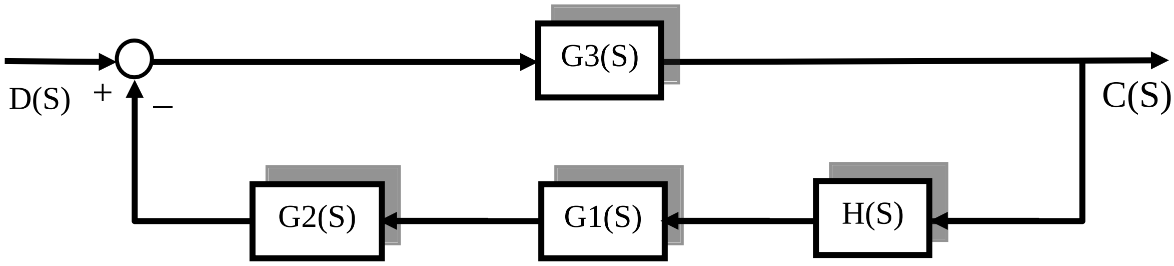
Example (17):

For the control system shown below, Obtain the transfer function $C(s)/R(s)$ and C(s)/D(s), then find an expression for the system response C(s).



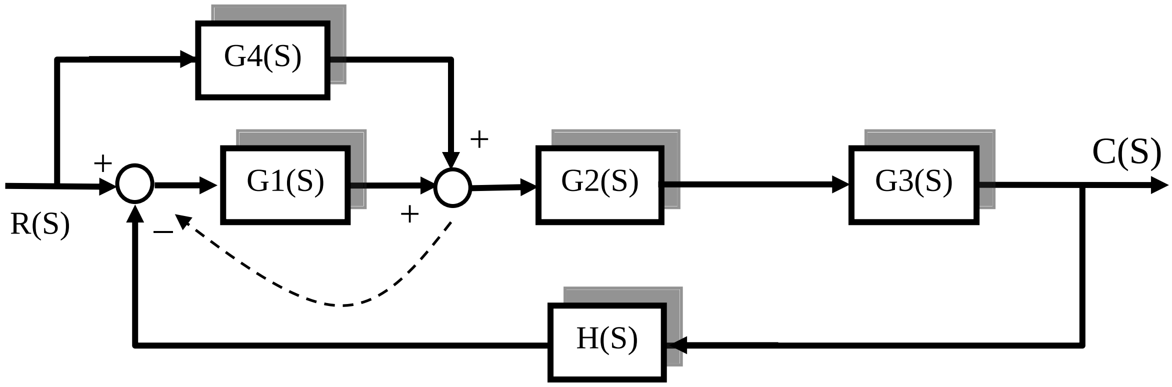
Using super position

Assume R(S) = 0, and rearrange the block diagram as follows:



$$\frac{C(S)}{D(S)} = \frac{G3(S)}{1 + G1(S)G2(S)G3(S)H(S)}$$

Now, assume D(S) = 0, rearrange the block diagram as follows



After moving the summing point as shown by the arrow indicated, the T.F. will be

$$\frac{C(S)}{R(S)} = \frac{G1(S)G2(S)G3(S)\left(1 + \frac{G4(S)}{G1(S)}\right)}{1 + G1(S)G2(S)G3(S)H(S)}$$

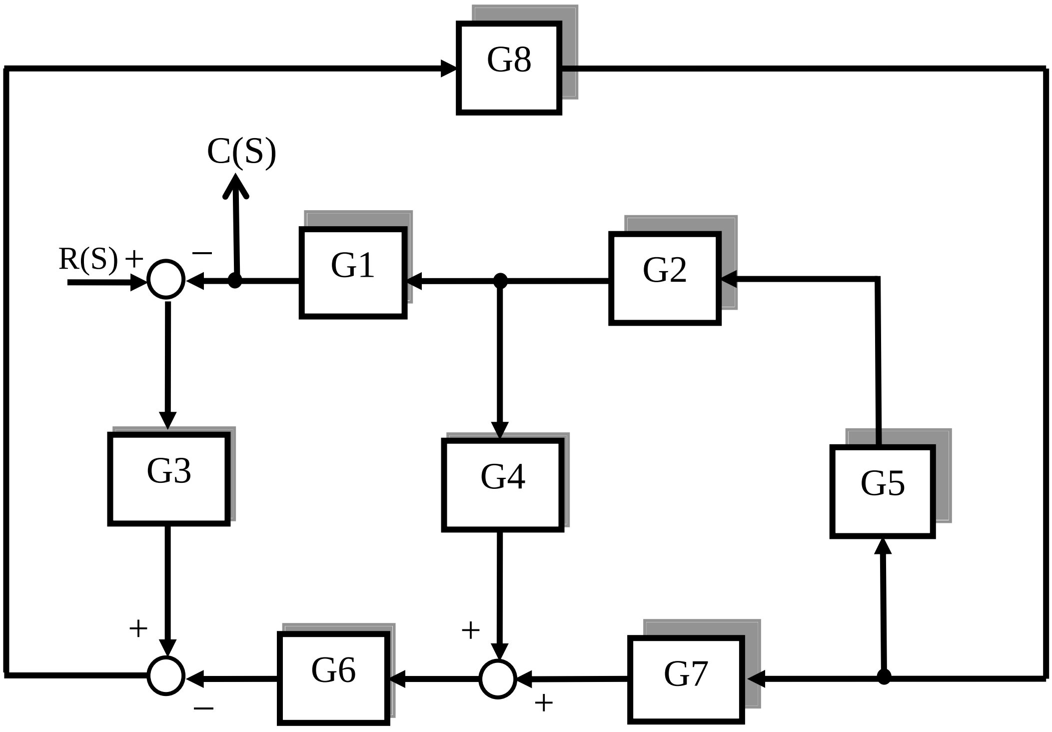
From both T.F's we can obtain the expression for the system response C(s) as:

$$C(S) = \frac{G3(S)\,D(S)}{1 + G1(S)G2(S)G3(S)H(S)} + \frac{G1(S)G2(S)G3(S)\left(1 + \frac{G4(S)}{G1(S)}\right)R(S)}{1 + G1(S)G2(S)G3(S)H(S)}$$

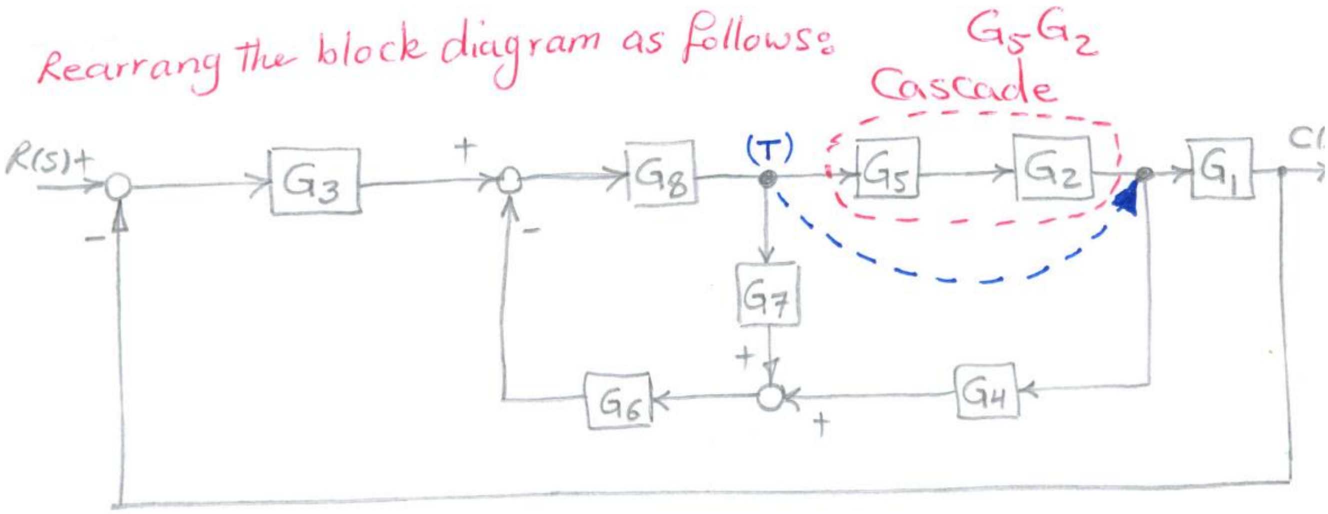
$$C(S) = \frac{G3(S)\,D(S) + \{G1(S)G2(S)G3(S) + G2(S)G3(S)G4(S)\}R(S)}{1 + G1(S)G2(S)G3(S)H(S)}$$

Example (18):

Simplify the block diagram shown below and then obtain the closed–loop transfer function $C(s)/R(s)$.

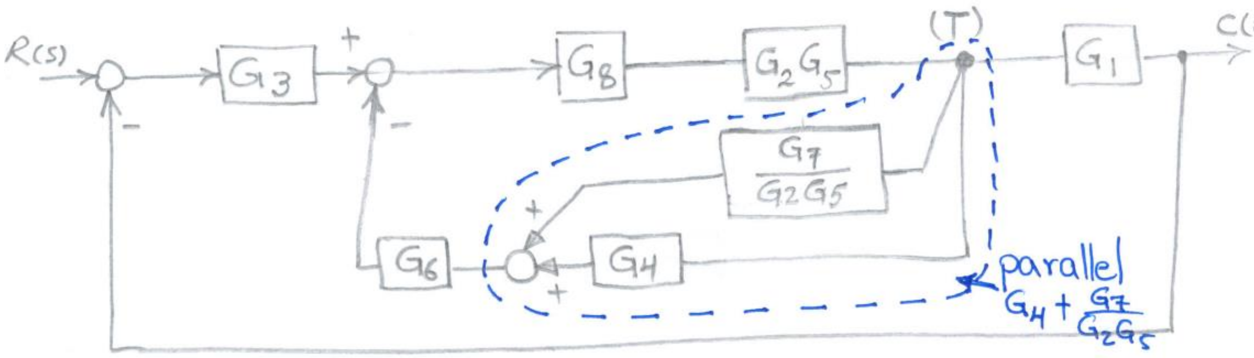


Rearrang the block diagram as follows:



* blocks G_5 & G_2 are Cascaded → G_2G_5

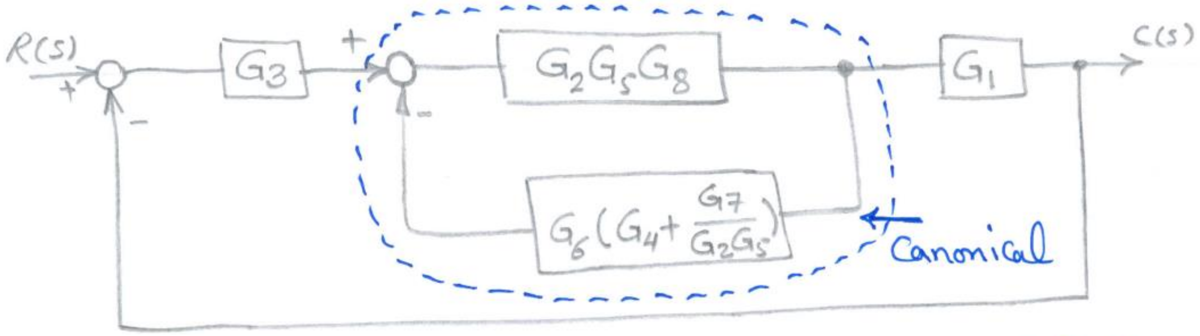
* Moving the take off point (T) in the forward direction as shown



* blocks G_4 & $\frac{G_7}{G_2 G_5}$ are in parallel $\Rightarrow G_4 + \frac{G_7}{G_2 G_5}$

* blocks $\left(G_4 + \frac{G_7}{G_2 G_5}\right)$ & G_6 are Cascaded $\Rightarrow G_6\left(G_4 + \frac{G_7}{G_2 G_5}\right)$

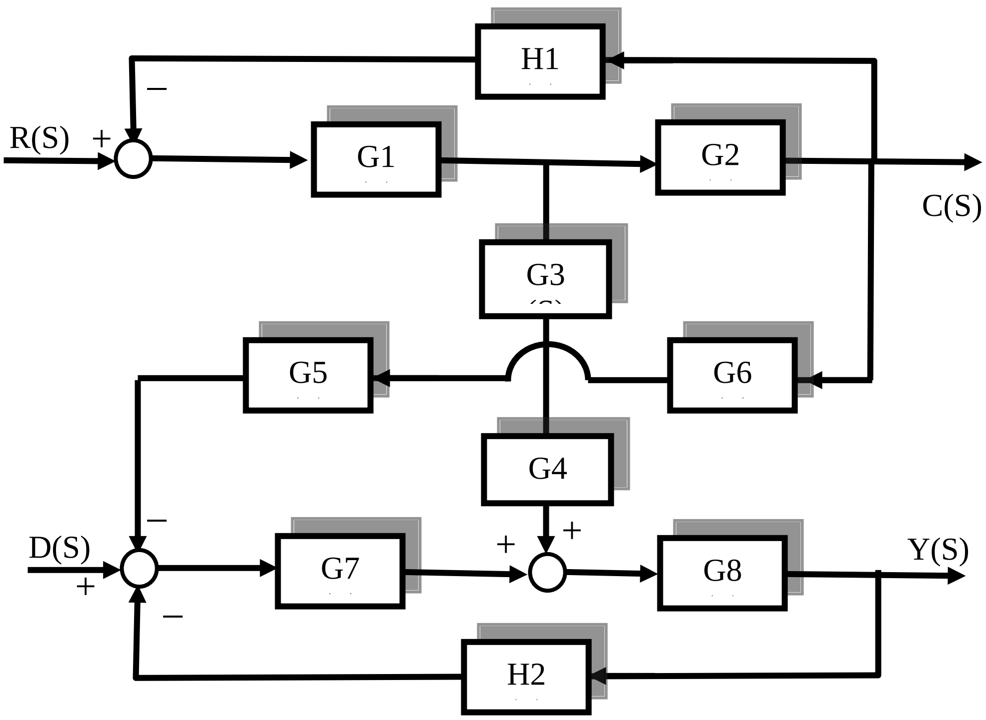
* blocks G_8 & $G_2 G_5$ are Cascaded $\Rightarrow G_2 G_5 G_8$



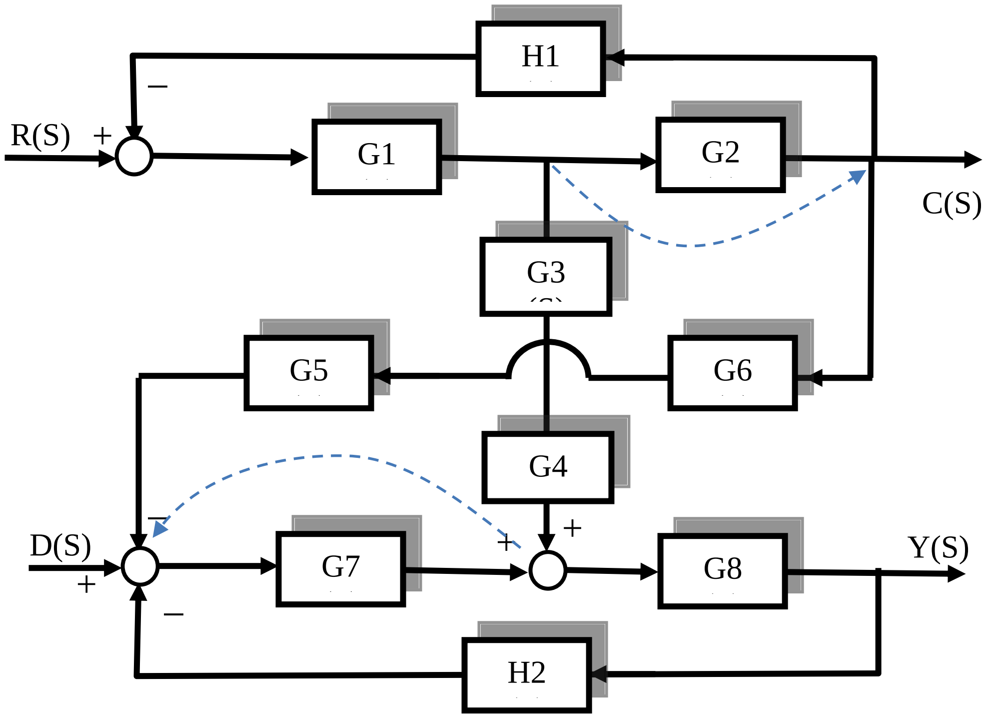
* blocks $G_2 G_5 G_8$ & $G_6\left(G_4 + \frac{G_7}{G_2 G_5}\right)$ are Canonical $= \frac{G_2 G_5 G_8}{1 + G_2 G_5 G_8 G_6\left(G_4 + \frac{G_7}{G_3 G_5}\right)}$

$$= \frac{G_2 G_5 G_8}{1 + G_2 G_4 G_5 G_6 G_8 + G_6 G_7 G_8}$$

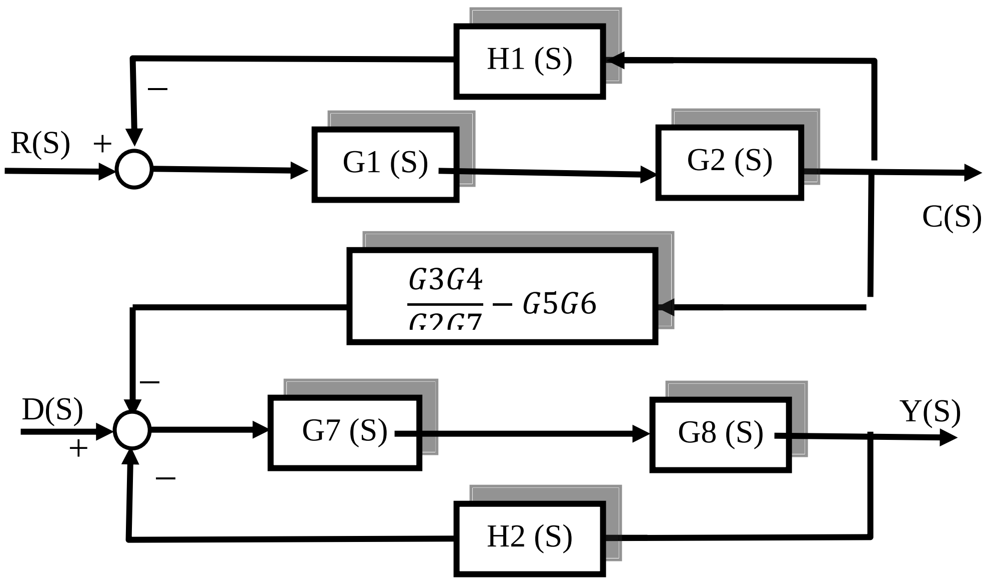
Example (19): For the Multi Input Multi Output (MIMO) control system shown below, find the total value of C(S) and Y(S).

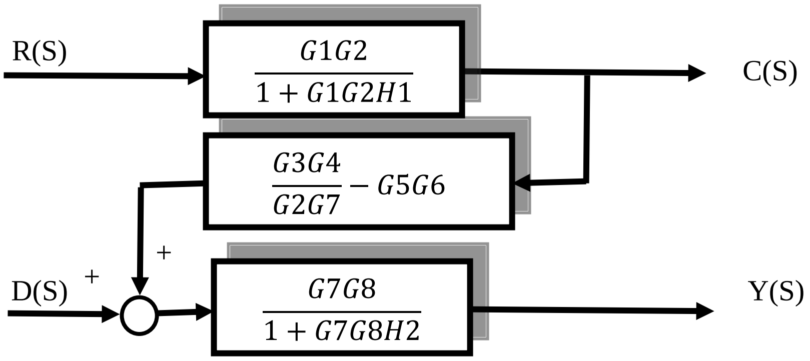


Moving the take off point in the forward direction (divide by G2)



Also move the summing point in the feedback direction (divide by G7)
The obtained branch (G3G4/G2G7) is in parallel with G5G6





If Y(S) = 0

Let the input R(S) = 0

$$\frac{C(s)}{D(s)} = 0$$

Let the input D(S) = 0

$$\frac{C(s)}{R(s)} = \frac{G_1G_2}{1 + G_1G_2H_1}$$

If C(S) = 0

Let the input R(S) = 0

$$\frac{Y(s)}{D(s)} = \frac{G_7G_8}{1 + G_7G_8H_2}$$

Let the input D(S) = 0

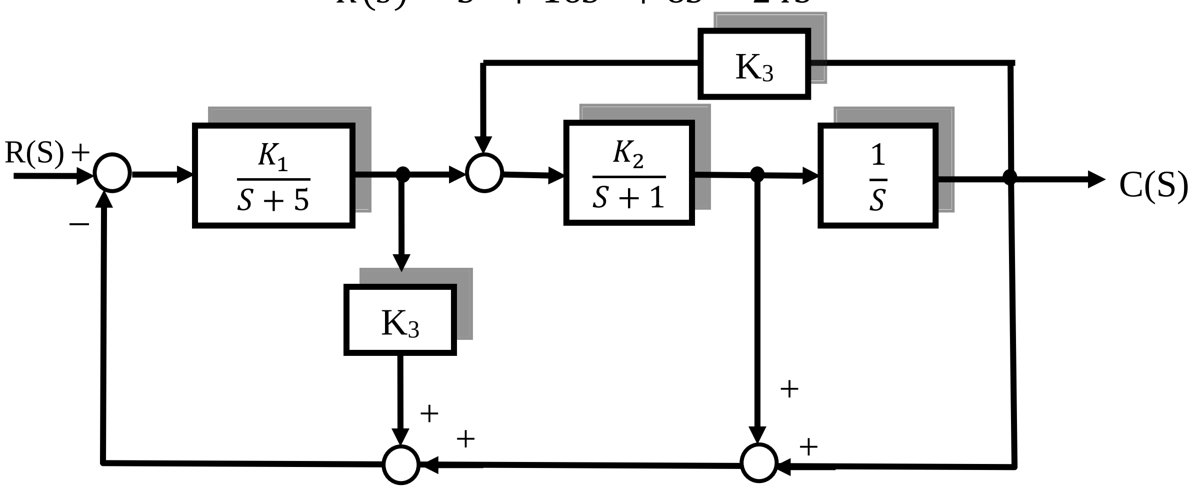
$$\frac{Y(s)}{R(s)} = \frac{G_1G_2}{1 + G_1G_2H_1} \times \frac{G_7G_8}{1 + G_7G_8H_2} \times \left\{\frac{G_3G_4}{G_2G_7} - G_5G_6\right\}$$

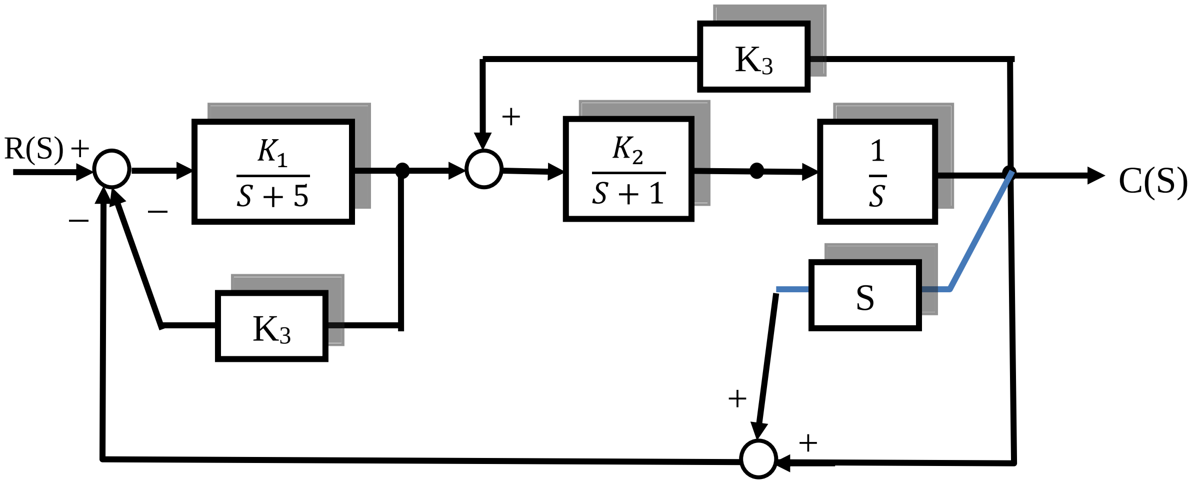
$$\frac{Y(s)}{R(s)} = \frac{G_1G_8(G_3G_4 - G_2^2G_5G_6G_7^2)}{(1 + G_1G_2H_1)(1 + G_7G_8H_2)}$$

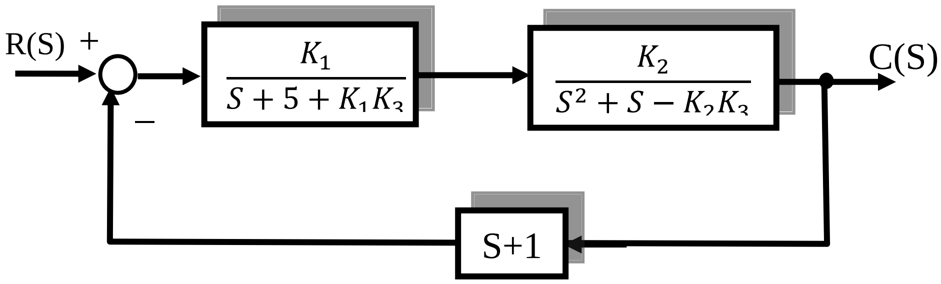
Example (20):

For the control system shown below, find the value of K_1, K_2 and K_3 if the system transfer function is

$$\frac{C(s)}{R(s)} = \frac{10}{S^3 + 16S^2 + 8S - 245}$$







$$\frac{C(s)}{R(s)} = \frac{K_1K_2}{(S+5+K_1K_3)(S^2+S-K_2K_3)+K_1K_2(S+1)}$$

$$\frac{C(s)}{R(s)} = \frac{K_1K_2}{S^3+S^2-K_2K_3S+5S^2+5S-5K_2K_3+K_1K_3S^2+K_1K_3S-K_1K_2K_3K_3+K_1K_2S+K_1K_2}$$

$$\frac{C(s)}{R(s)} = \frac{K_1K_2}{S^3+S^2(K_1K_3+6)+S(K_1K_2+K_1K_3-K_2K_3+5)+K_1K_2-K_1K_2K_3K_3-5K_2K_3}$$

By Comparing:

$$K_1K_2 = 10 \quad (1)$$

$$K_1K_3 + 6 = 16 \rightarrow\rightarrow K_1K_3 = 10 \quad (2)$$

$$From\ (1)\&\ (2)\ K_2 = K_3$$

$$K_1K_2 + K_1K_3 - K_2K_3 + 5 = 8 \quad \rightarrow \quad K_2K_3 = 10+10+5-8 \quad \rightarrow \quad K_2K_3 = 17 \quad (3)$$

$$from\ (3) \quad K_2 = K_3 = \sqrt{17} = 4.12311$$

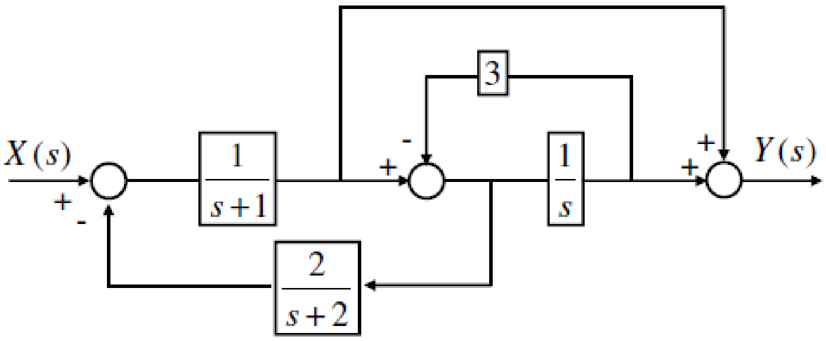
$$from\ (1)\ K_1 = \frac{10}{\sqrt{17}} = 2.42536$$

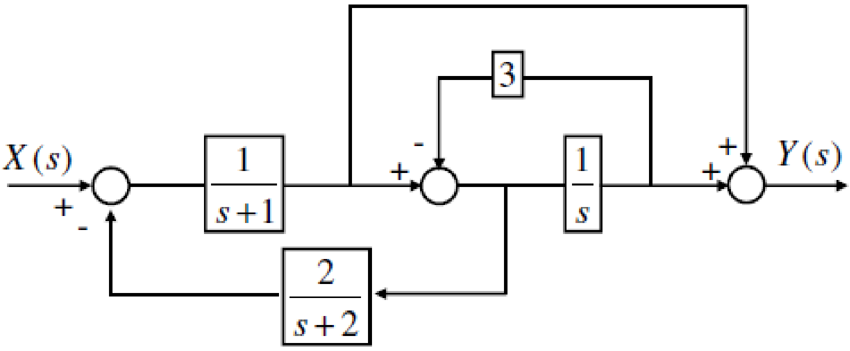
As Check:

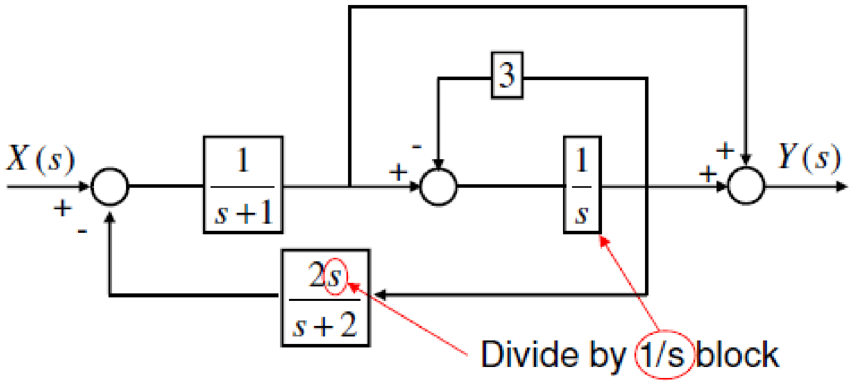
$$K_1K_2 - K_1K_2K_3K_3 - 5K_2K_3 = 10 - 10\times 17 - 5\times 17 = -245$$

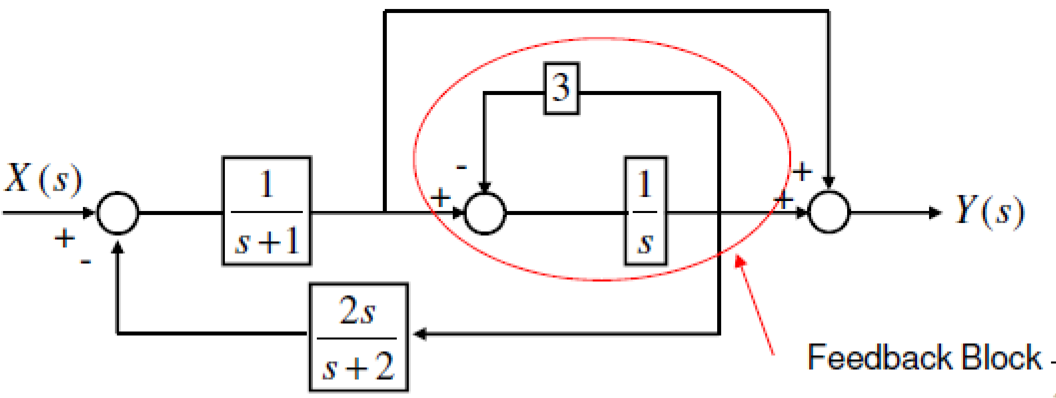
Example (21):

Find the system T.F.

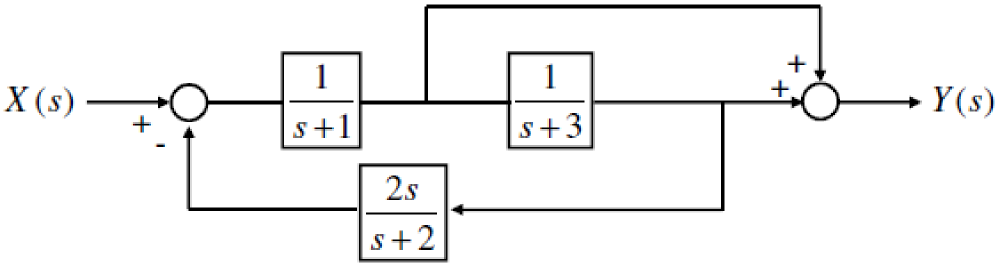


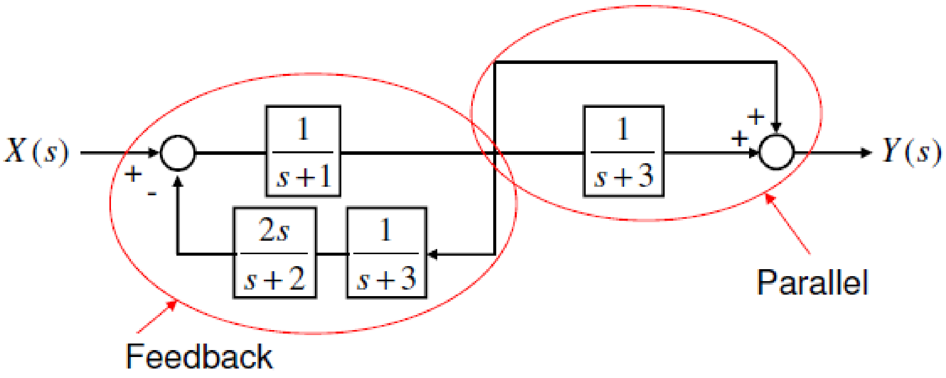


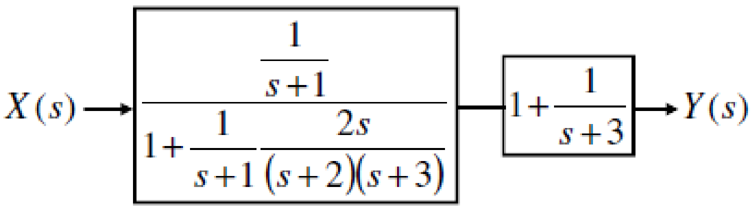


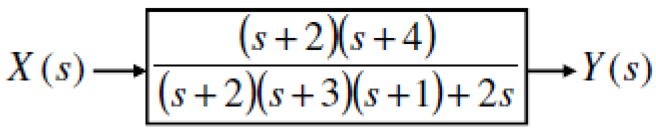


$$\frac{\frac{1}{s}}{1+\left(\frac{1}{s}\right)(3)} = \frac{1}{s+3}$$



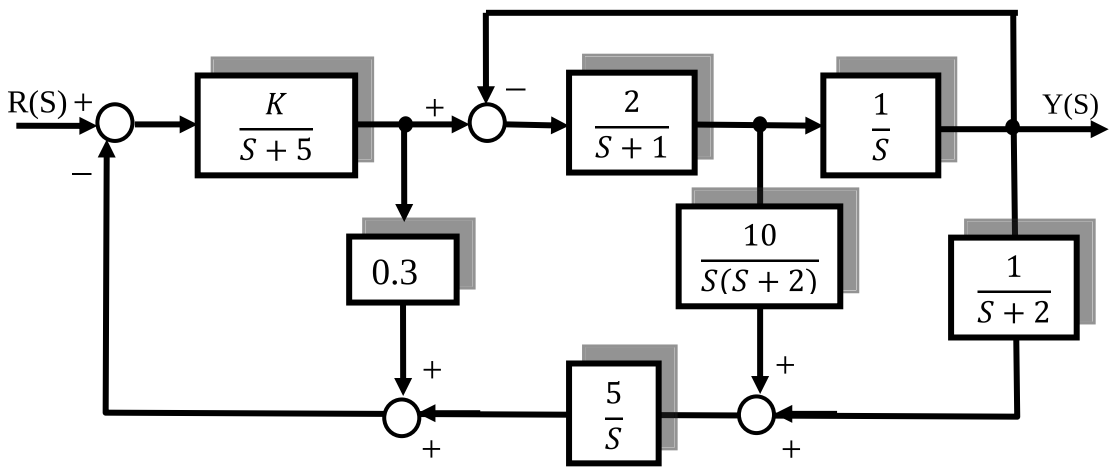




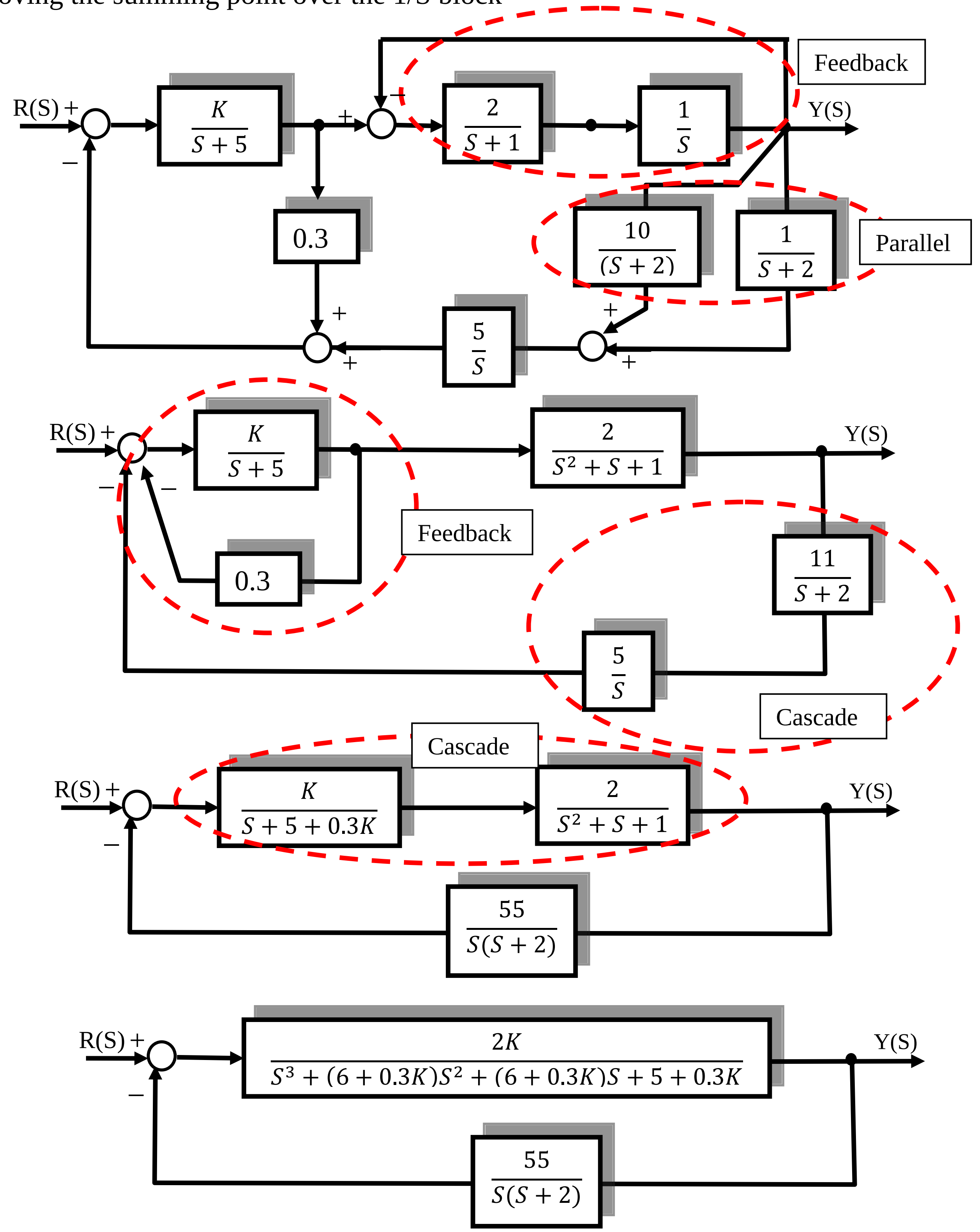


Example (22):

Find the Transfer Function of the control system described by the block diagram given below.

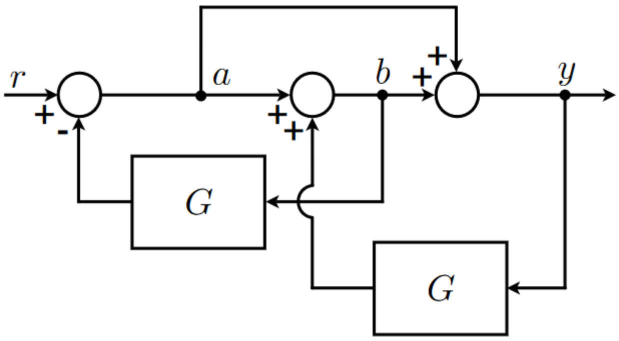


Moving the summing point over the 1/S block



$$\frac{Y(S)}{R(S)} = \frac{2KS(S+2)}{S(S^3+(6+0.3K)S^2+(6+0.3K)S+5+0.3K)(S+2)+110K}$$

Example (23):



The best way to solve this problem is by writing a system of equations involving the signals a and b. It is probably impossible (or very difficult) to solve this problem using only block diagram manipulations, whereas the use of Mason's rule is possible but overly complicated.

Equations involving the signals a and b:

$$a = r - Gb \quad (1)$$
$$b = a + Gy \quad (2)$$
$$y = a + b \quad (3)$$

Plugging (2) in (1) and (3), we have

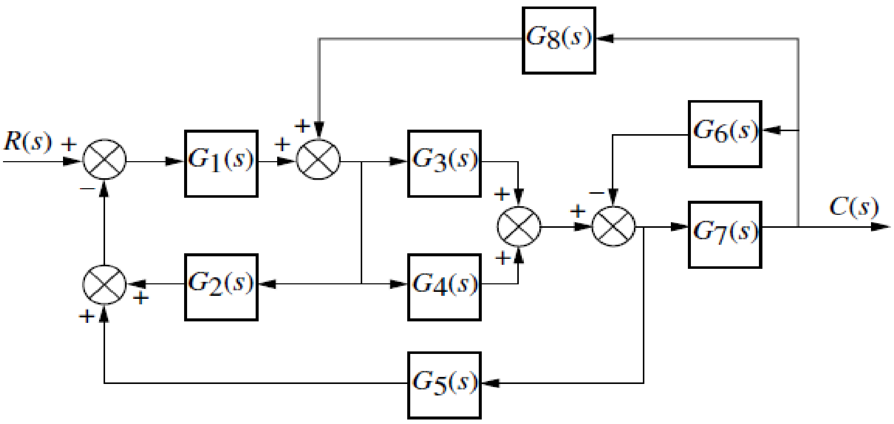
$$a = r - Ga - G^2 y$$
$$y = 2a + Gy$$

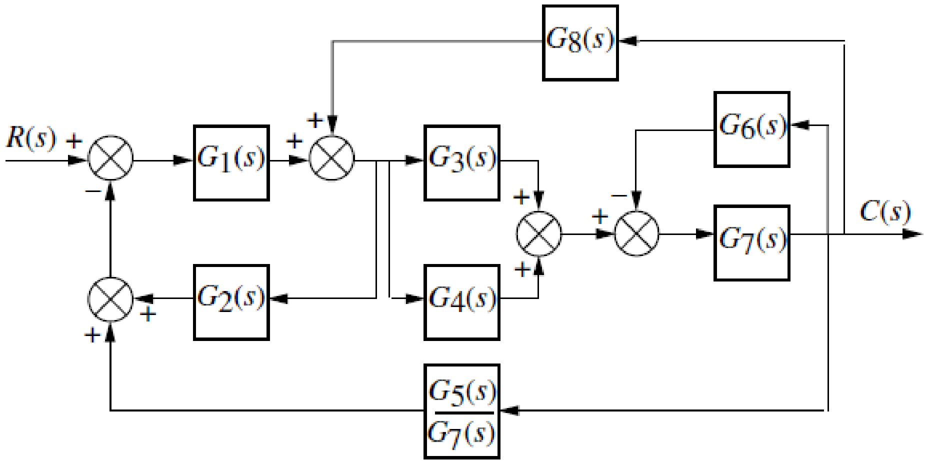
which lead to

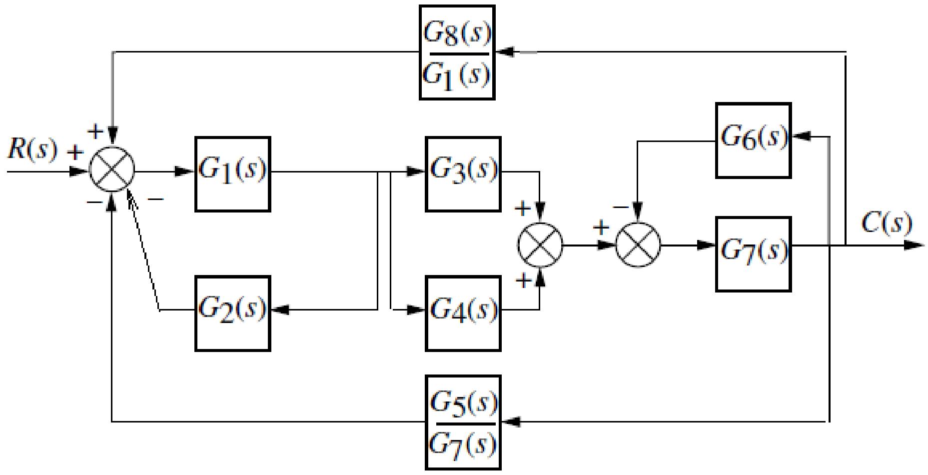
$$y = 2\frac{r - G^2 y}{1+G} + Gy = \frac{2}{1+G}r + \frac{G - G^2}{1+G}y$$
$$\Rightarrow \left(1 - \frac{G-G^2}{1+G}\right) y = \frac{2}{1+G}r$$
$$\Rightarrow \frac{y}{r} = \frac{2}{1+G^2}$$

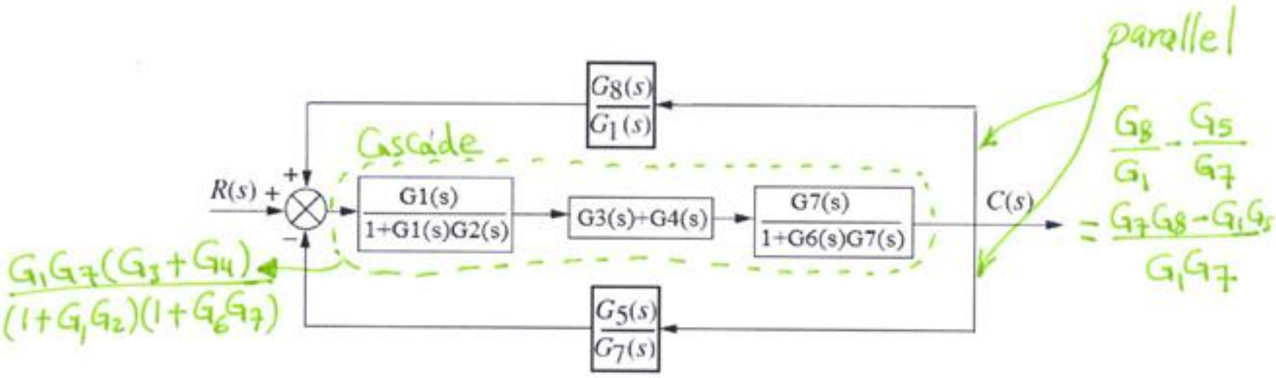
Example (24):

Find the transfer function for the control system given below.









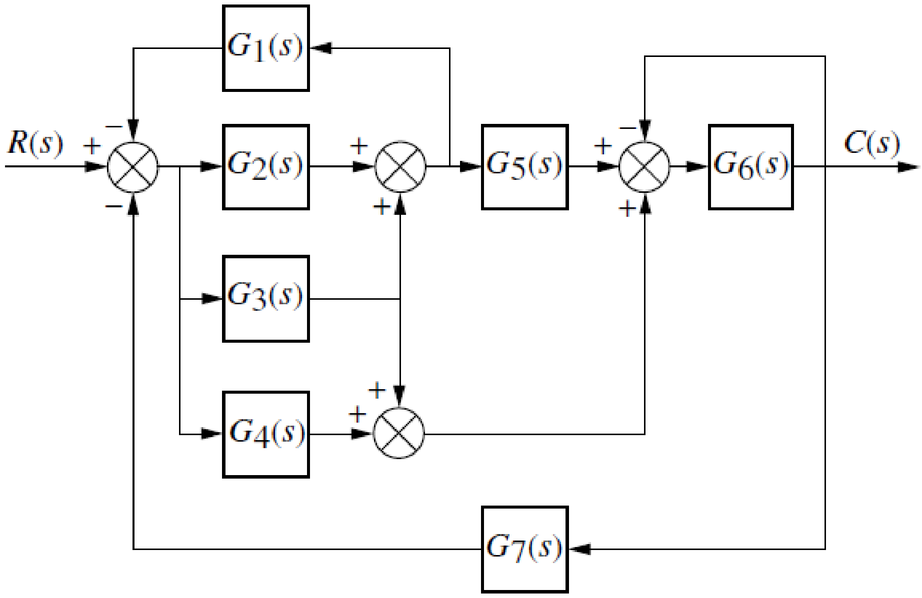
$$\frac{C(s)}{R(s)} = \frac{G_1^2 G_7^2 (G_3+G_4)}{G_1G_7(1+G_1G_2)(1+G_6G_7) - G_1G_7(G_3+G_4)(G_7G_8-G_1G_5)}$$

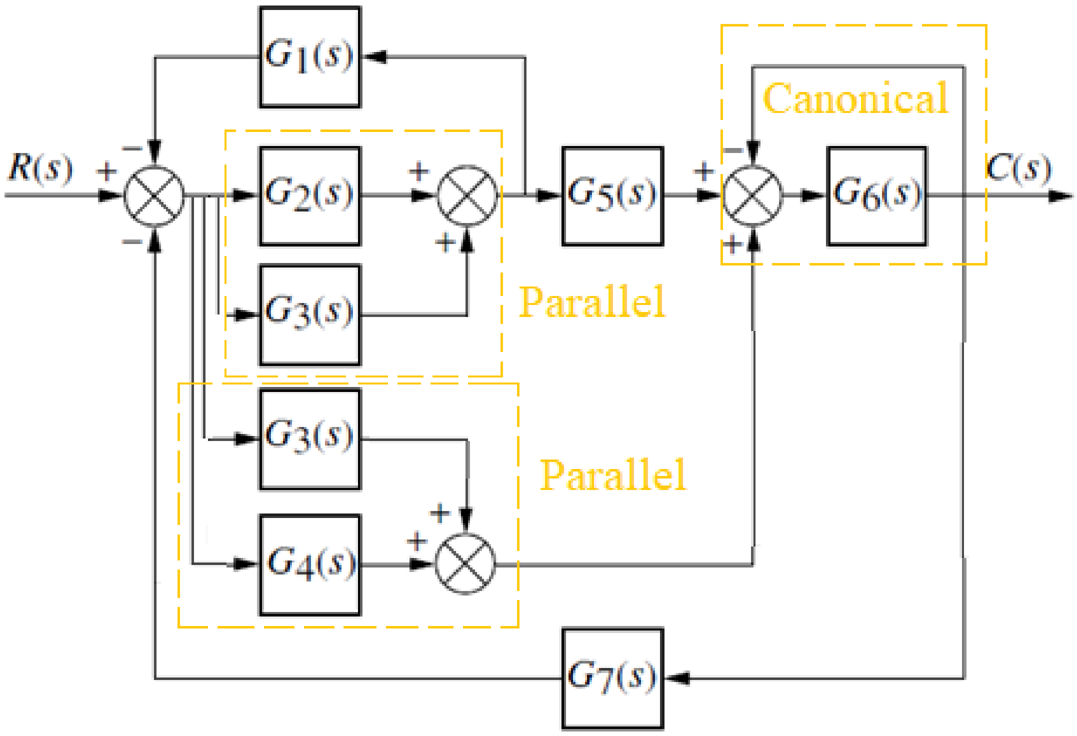
$$\frac{C(s)}{R(s)} = \frac{G_1G_7(G_3+G_4)}{(1+G_1G_2)(1+G_6G_7) - (G_3+G_4)(G_7G_8-G_1G_5)}$$

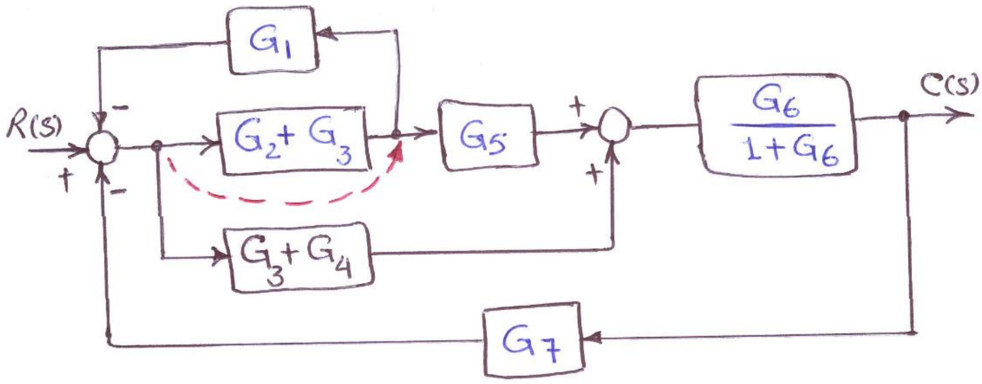
$$\frac{C(s)}{R(s)} = \frac{G_1G_3G_7 + G_1G_4G_7}{1 + G_1G_2 + G_6G_7 + G_1G_2G_6G_7 - G_3G_7G_8 - G_4G_7G_8 + G_1G_3G_5 + G_1G_4G_5} \quad \#$$

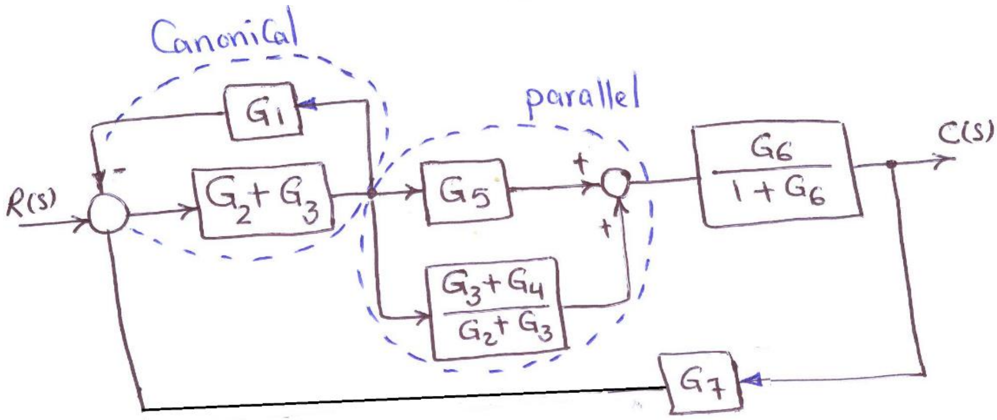
Example (25)

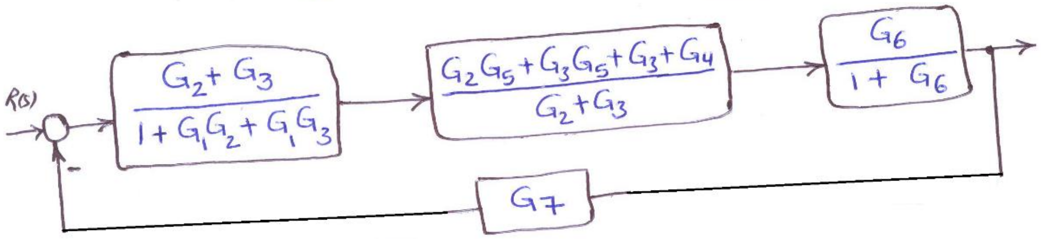
Consider the control system shown below, find the system transfer function.






G1
R(s)
G2+ G3
G5
G6 / 1+G6
C(s)
G3+G4
G7

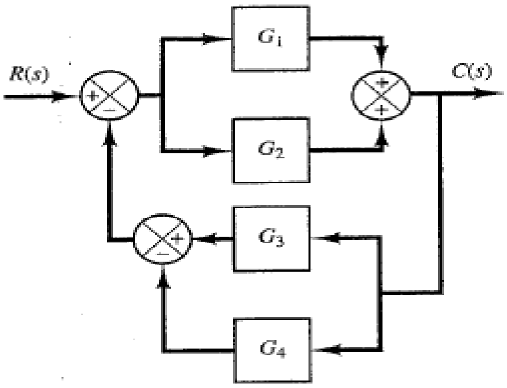

Canonical
parallel
G1
R(s)
G2+ G3
G5
G6 / 1+G6
C(s)
G3+G4 / G2+ G3
G7


R(s)
G2+ G3 / 1+ G1G2+ G1G3
G2G5+G3G5+G3+G4 / G2+G3
G6 / 1+ G6
G7

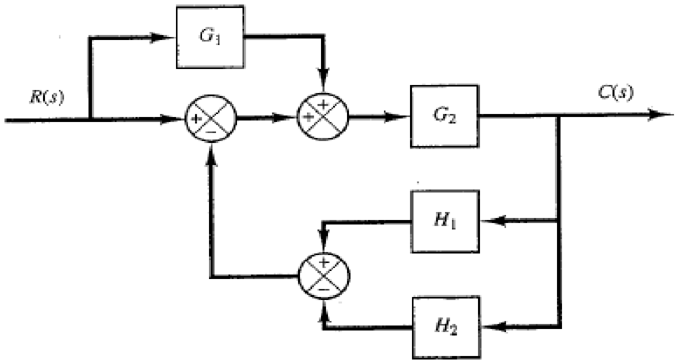
Sheet 2 (Block Diagram)

Problem #1

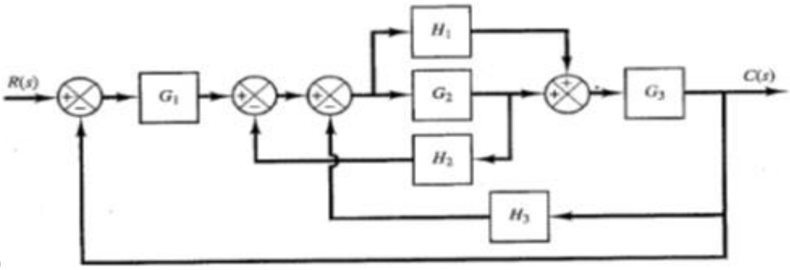
Simplify the following control systems using block diagram algebra, and then find the transfer function *C*(s) / *R*(s).



(a)



(b)

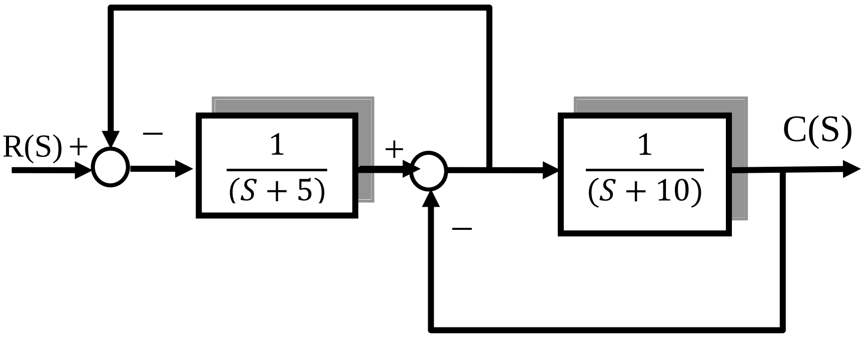


c)

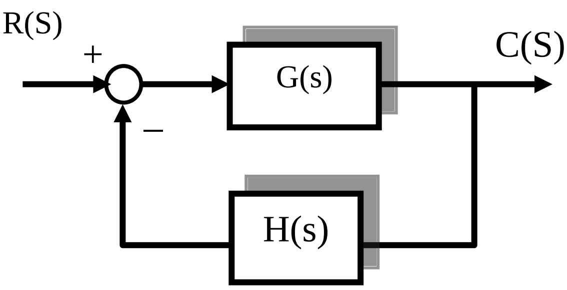
Problem #2

For the control system shown in Fig. (b) below,

a) Determine G(s) and H(s) that are equivalent to the block diagram of fig. (a)
b) Determine the transfer function C(s)/R(s)



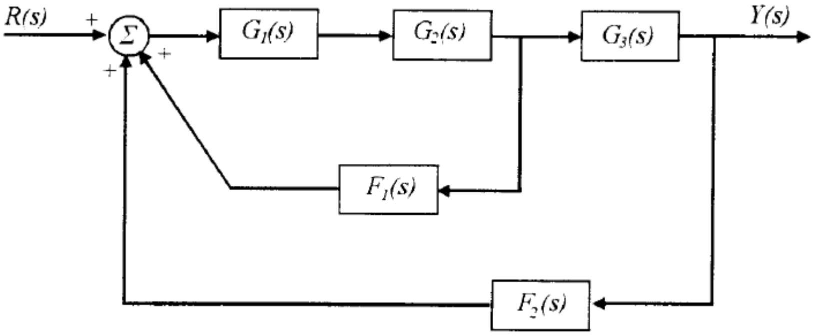
(a)



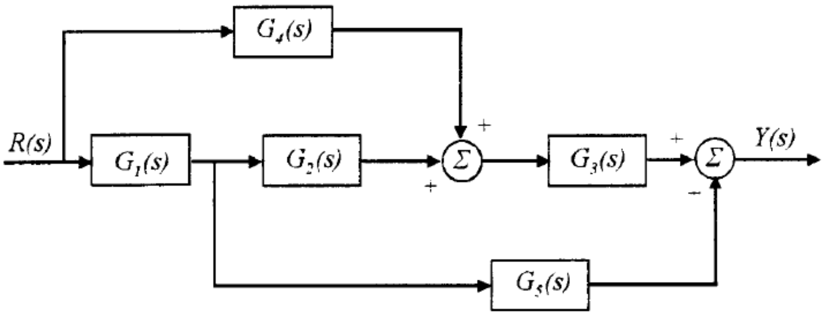
(b)

Problem #3

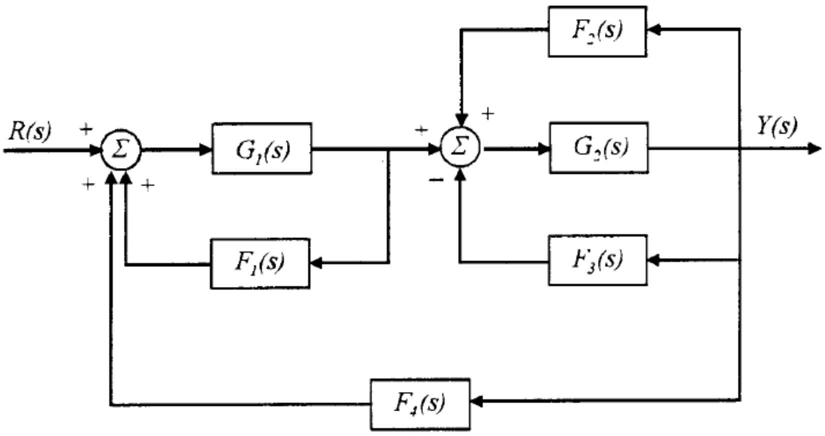
Simplify the following control systems using block diagram algebra, and then find the transfer function *Y*(s) / *R*(s).



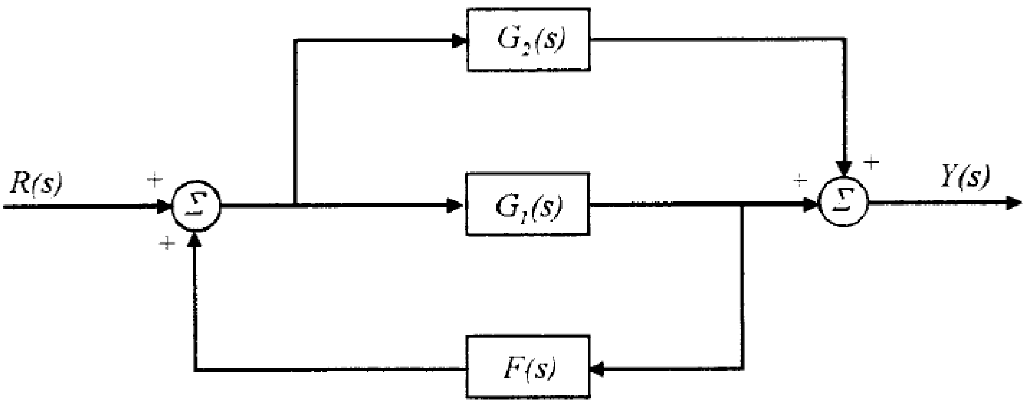
(a)



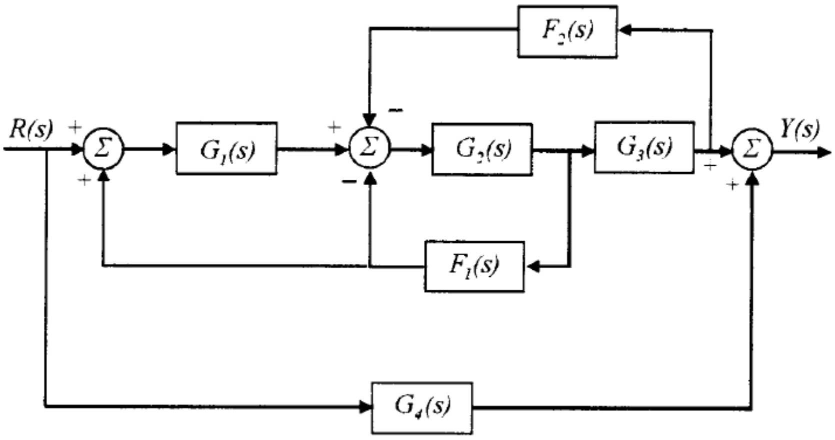
(b)



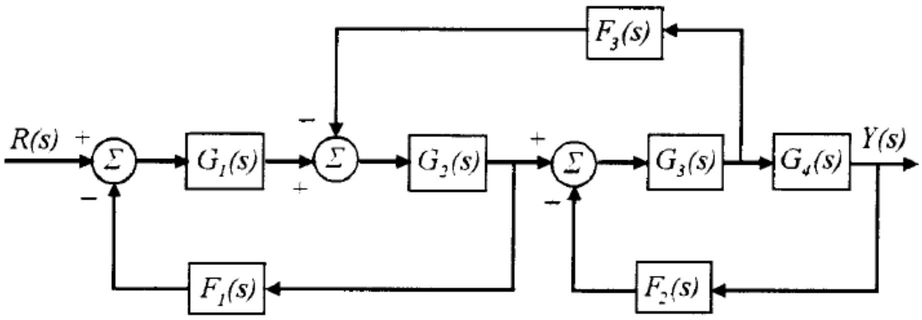
(c)



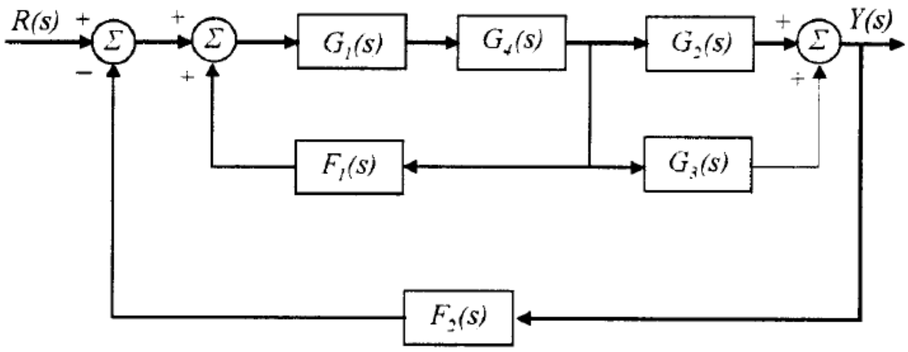
(d)



(e)



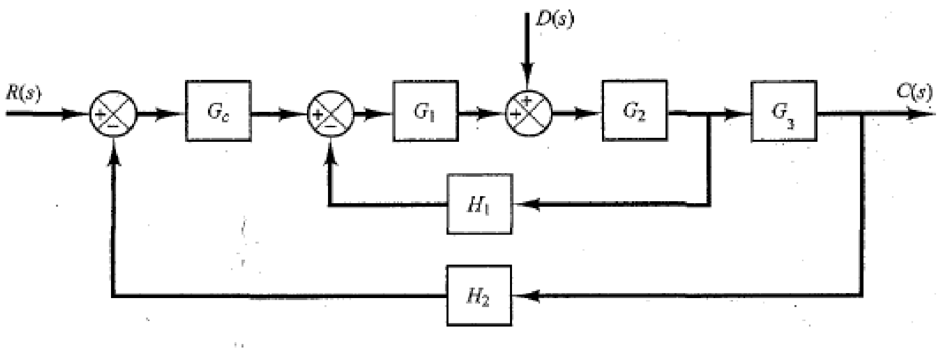
(f)

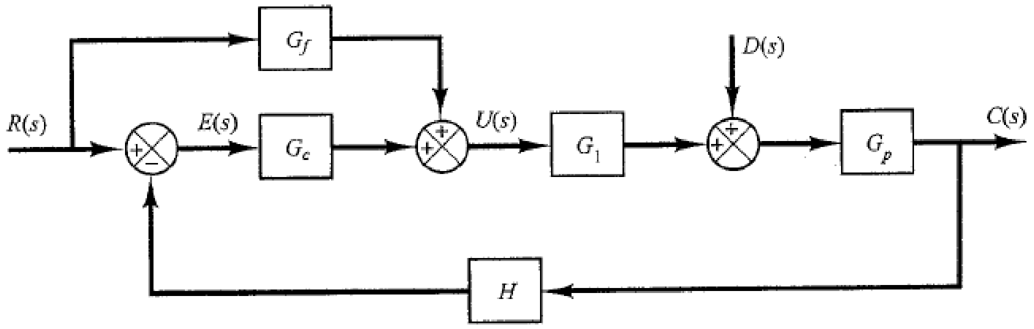


(g)

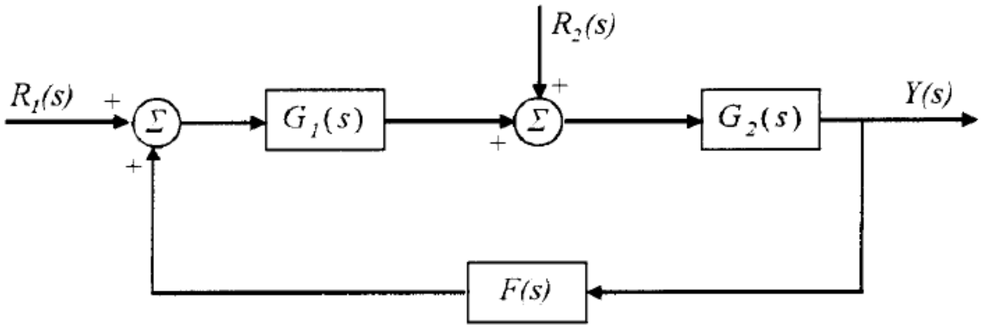
Problem #4

Obtain the transfer functions *C(s)/R(s)* and *C(s)/D(s)* of the systems shown below



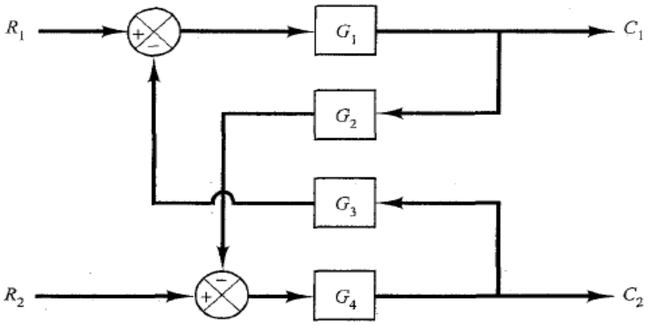


Obtain the transfer functions *Y(s)/R1(s)* and *Y(s)/R2(s)* of the system shown below



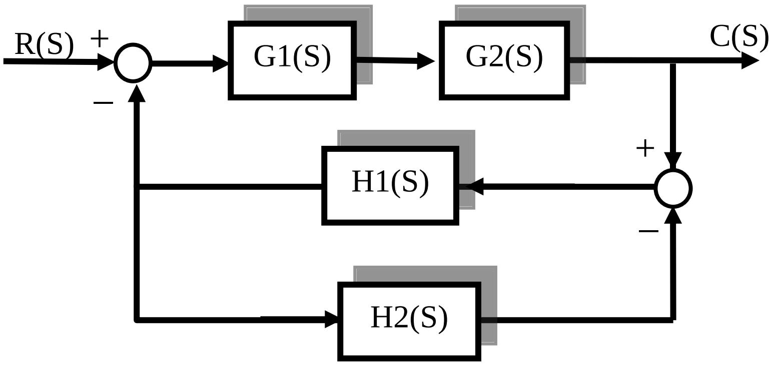
Problem #5

The control system, shown in Fig. below, has two inputs and two outputs. Find $C_1(s)/R_1(s)$, $C_1(s)/R_2(s)$, $C_2(s)/R_1(s)$ and $C_2(s)/R_2(s)$.



Problem #6

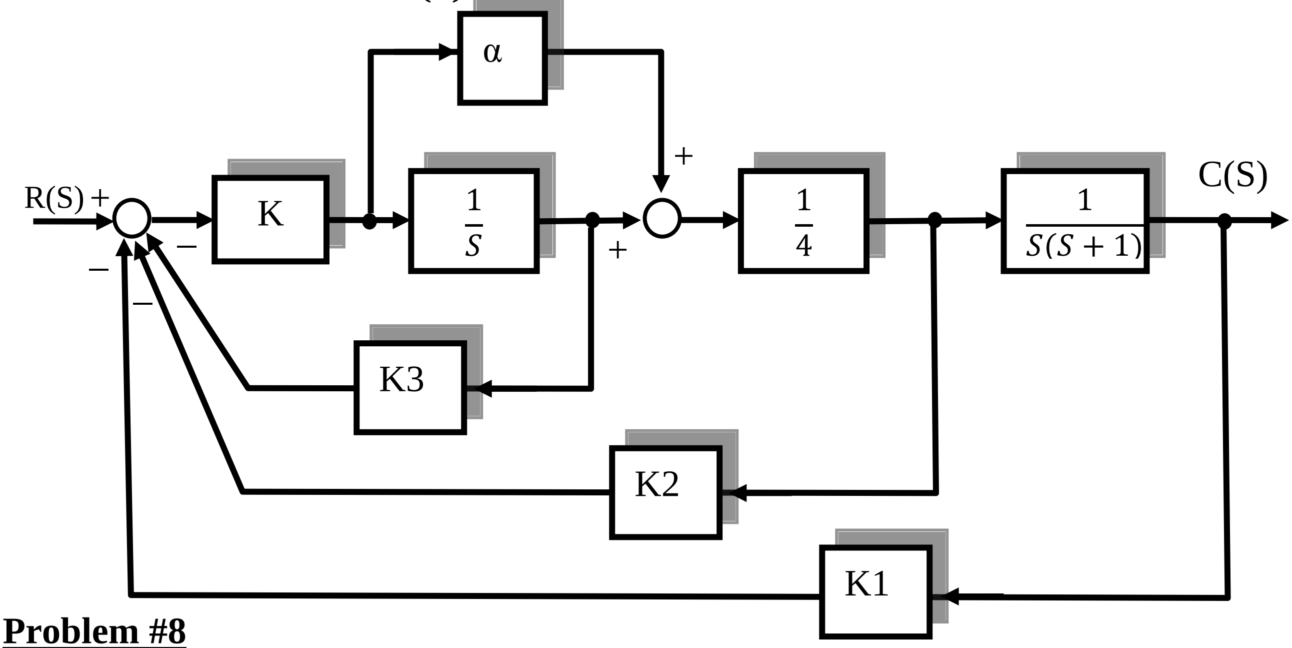
For the control system, shown in figure below, obtain the system transfer function.



Problem #7

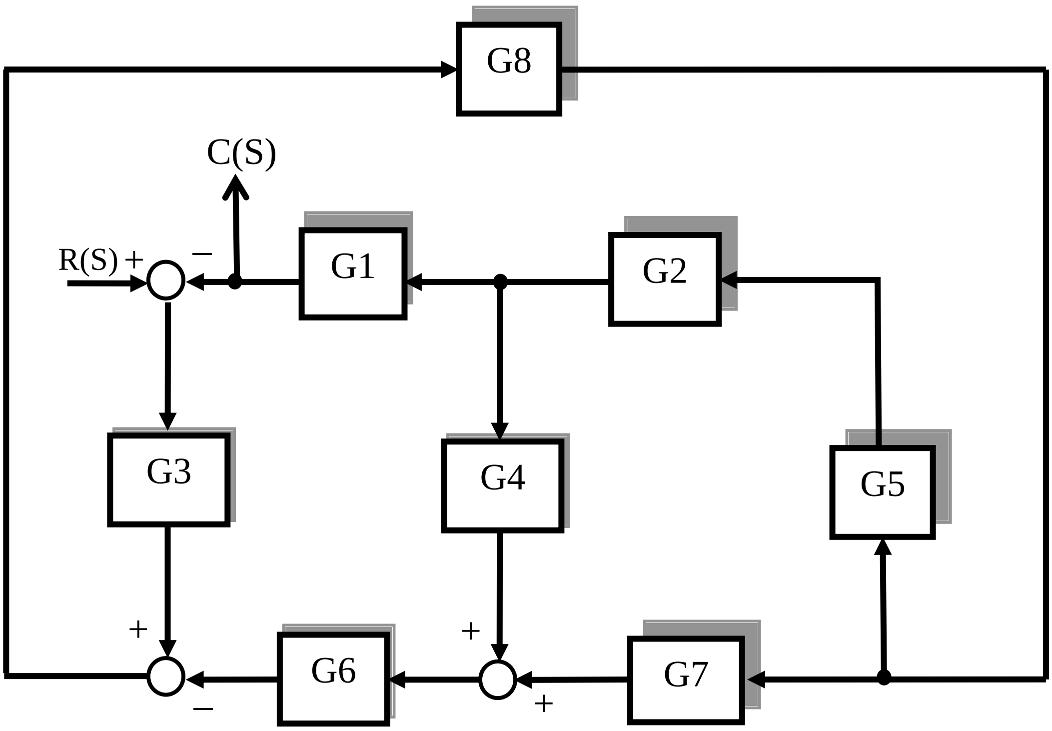
For the control system shown below, find α, K, K1, K2 and K3 if Known that

$$\frac{C(S)}{R(S)} = \frac{10\,(S+1)}{S^3 + 3\,S^2 + 12\,S + 10}$$



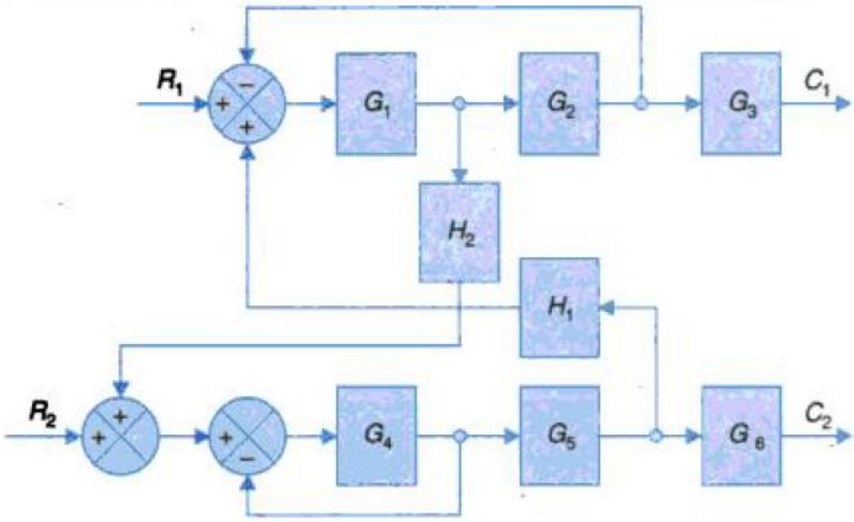
Problem #8

Simplify the block diagram shown below and then obtain the closed–loop transfer function $C(s)/R(s)$.



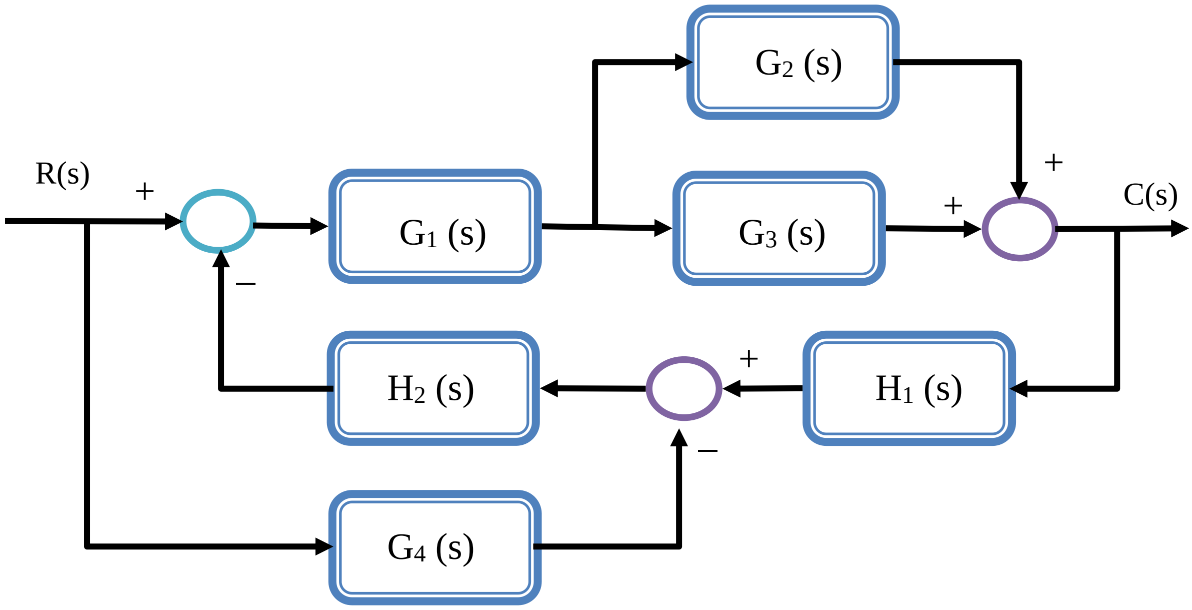
Problem #9

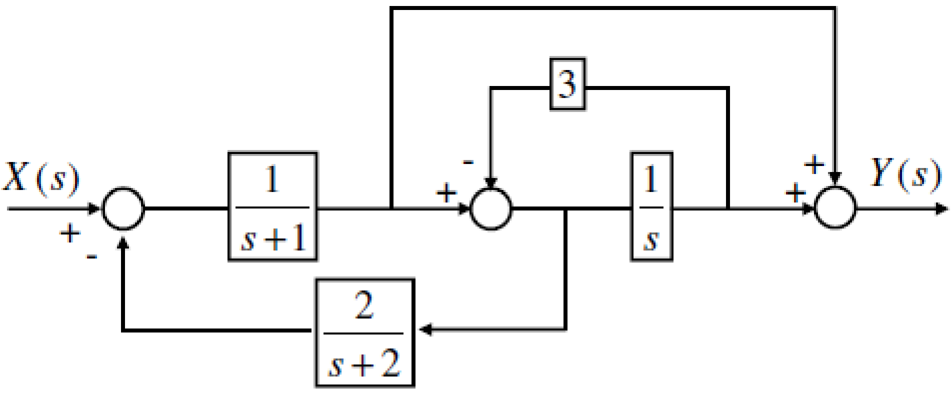
For the MIMO control system shown below, find the transfer matrix.



Problem #10

For the control systems shown below, find the transfer function.





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[2] Franklin, G. F., Powell, J. D., and Emami-Naeini, A. Feedback Control of Dynamic Systems. Addison-Wesley, Reading, MA, 1986.

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